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# Peer Effects in Networks: a Survey

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# ABSTRACT

We survey the recent, fast-growing literature on peer effects in networks. An important recurring theme is that the causal identification of peer effects depends on the structure of the network itself. In the absence of correlated effects, the reflection problem is generally solved by network interactions even in non-linear, heterogeneous models. By contrast, microfoundations are generally not identified. We discuss and assess the various approaches developed by economists to account for correlated effects and network endogeneity in particular. We classify these approaches in four broad categories: random peers, random shocks, structural endogeneity and panel data. We review an emerging literature relaxing the assumption that the network is perfectly known. Throughout, we provide a critical reading of the existing literature and identify important gaps and directions for future research.



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# 1 Introduction

Human beings are by nature social animals. We are affected by others on many dimensions and in many contexts. As parents, we bear witness to our influence and to the influence of teachers, friends and classmates on our children's language, beliefs and behaviors. As academic researchers, we interact frequently and these interactions affect our research, collaborations and technology adoption. Most empirical studies of peer effects indeed document strong positive correlations between individual and peers' outcomes. Personal experience and correlation are, of course, not causation and researchers who wish to credibly assess the causal effect of peers face some formidable econometric challenges. Correlation in outcomes among peers may be caused by correlated unobserved characteristics due, for instance, to endogenous choice of peers or to common shocks. This is the central problem of *correlated effects*. Even when this problem has been addressed, however, distinguishing between the impact of peers' outcomes (endogenous peer effect) and of peers' characteristics (contextual peer effects) may be impossible because of simultaneity in the behavior of interacting agents. This is the *reflection problem*, clarified by Manski (1993). Overall, the combination of widespread indirect evidence and methodological challenges has given rise to an immense and still expanding literature, which spans all the different realms of applied analysis in economics and in other social sciences.[1]

The analysis of peer effects has recently expanded in a new direction, stimulated by a fast-growing interest in networks. Historically, and because of data availability, many studies of peer effects considered peer groups which partition the population, such as classrooms or villages. Agents generally have distinct sets of peers, however, leading to *a network of interactions*. How do network interactions affect the analysis of peer effects and the treatment of correlated effects and of the reflection problem? About 15 years ago, four studies independently understood that the reflection problem is naturally solved by network interactions (Bramoullé, Djebbari, and Fortin, 2009; De Giorgi, Pellizzari, and Redaelli, 2010; Lin, 2010; Laschever, 2011). In Bramoullé, Djebbari, and Fortin (2009), we characterize the identification conditions of the benchmark linear-in-means model of peer effects when agents interact through a network. We show that once the problem of correlated effects has been addressed, endogenous and contextual peer effects are identified under intransitivity, when peers of peers are not peers. Their characteristics then have an impact on individual outcome only through their effect on peers' outcome, providing valid instruments. An important insight is that identification depends on the structure of the network itself and exploits holes in the structure.

[1] In economics, see Manski (2000), Epple and Romano (2011) and Sacerdote (2011) on education, Graham (2018) on neighborhood effects, and Angrist (2014) for a critical review. In sociology and social psychology, see for instance Alexander et al. (2001) on smoking, Salmivalli (2010) on bullying, and Kreager, Rulison, and Moody (2011) on delinquency.

Since then, the literature on peer effects in networks has grown quickly and in many directions. In this survey, our objectives are to provide a structured discussion of this evolving literature, to critically assess its achievements and to identify important gaps and natural directions for future research. Doing so, we develop a number of novel formal and informal arguments. The survey, of course, does not pretend to be exhaustive. Given space constraints, we decided to focus on intuition and main identification issues in the empirical analysis of peer effects.[2]

We do not cover, in particular, a large theoretical literature on social interactions where researchers explicitly model how agents' choices depend on their peers. Studies of games played on networks are notably surveyed in Jackson and Zenou (2015) and Bramoullé and Kranton (2016). There are, of course, interesting connections between theoretical and empirical analyses of peer effects. In some contexts, econometric models of peer effects can be obtained as best responses of games played on networks. Well-designed theoretical models of social interactions can be structurally estimated, see Banerjee et al. (2013). We discuss the microfoundations of the empirical models of peer effects in Section 2.

The survey is organized in three parts. In Section 2, we start from the linear-in-means framework of Bramoullé, Djebbari, and Fortin (2009). Assume that the problem of correlated effects is fully solved: the network of interactions and observed characteristics are exogenous. Are endogenous and contextual peer effects identified despite simultaneity? We briefly restate the main results and insights of our original analysis. We show that the network-based identification strategy holds much more generally. We consider group interactions and relate the surprising identification results for groups to the broader study of group size. We discuss how identification can also be obtained by restrictions on the error terms. We then discuss the influential overview of Angrist (2014). We show that the counterexample proposed in Section 6 of Angrist (2014) simply illustrates a well-known case of non-generic identification failure. We finally clarify often misunderstood issues on mechanisms and microfoundations. Building on Blume et al. (2015), we show that the microfoundations underlying peer effect regressions are generally not identified, even under strong restrictions. An important implication is that contrary to widespread belief, the endogenous peer effect may not give rise to a social multiplier (Boucher and Fortin, 2016).

In Section 3, we focus on the central problem of correlated effects. Assume now that the network may be endogenous and that, more generally, correlated unobserved characteristics may affect outcomes. How can researchers account for correlated effects and causally identify peer effects when agents interact through networks? Researchers have employed diverse strategies to address this problem. We classify them in four broad categories: random peers, random shocks, structural

[2]Our survey complements other recent surveys on networks such as Fafchamps (2015), Boucher and Fortin (2016), de Paula (2017), and Advani and Malde (2018).

endogeneity and panel data. We clarify, first, the identification possibilities opened up by random assignment of peers. We show that endogenous peer effects are identified under network interactions through a variant of the peers of peers' idea. By contrast, we emphasize that with random peers the impact of peers' characteristics may not be identified because of a classical problem of omitted variables. Second, we consider a randomized treatment affecting outcome. We show that contextual and endogenous peer effects are identified with a randomized treatment in a linear-in-means framework even when the network is endogenous, if the network is not affected by the treatment. We discuss an emerging literature on randomized treatments and spillovers which relaxes parametric assumptions. Third, we review an active literature proposing structural frameworks to address correlated effects. A main idea here is to account for network endogeneity by explicitly modelling network formation and its connection to the peer effect regression. This literature has strong ties with the fast-growing econometric literature on network formation. Fourth, we discuss how panel data can help account for correlated effects. We observe that the analysis of peer effects in networks in a panel context is underdeveloped, both empirically and methodologically.

In Section 4, we review an emerging literature relaxing the assumption that the network of interactions is perfectly known. Imperfect knowledge of the network may arise because of sampling, measurement error and general uncertainty on the relevant peers. This literature is small but fast-growing and the first results are encouraging. Peer effects can, in principle, be identified even with very limited knowledge on the network. Much more research is needed, however, to better understand the implications of such imperfect knowledge. Section 5 briefly concludes.

# 2 Identification of peer effects through networks

**Bramoullé, Djebbari and Fortin, 2009**. A researcher has data on $n$ agents, their characteristics $\mathbf{x}$, their outcomes $\mathbf{y}$ and a binary directed network connecting the agents. Let $N_i$ denote the set of agents who affect $i$, of size $d_i = |N_i|$. Denote by $\mathbf{G}$ the following interaction matrix: $g_{ij} = 1/d_i$ if $j \in N_i$ and 0 otherwise. We assume that characteristics, outcomes and the network $(\mathbf{x}, \mathbf{y}, \mathbf{G})$ have been generated by a stochastic process.[3] In a *linear-in-means* model of peer effects, an agent's outcome depends on her own characteristics, her peers' characteristics and her peers' outcomes as follows.

[3] In a number of papers (e.g. Lee, Liu, and Lin, 2010), $\mathbf{G}$ is fixed (non-stochastic). One limitation of this assumption is that links between individuals are not allowed to be affected by a (policy) shock in $\mathbf{x}$, such as in Comola and Prina (2019).

Consider a unique characteristic and assume that no individual is isolated: $\forall i, d_i > 0$.

$$y_i = \alpha + \gamma x_i + \delta \frac{1}{d_i} \sum_{j \in N_i} x_j + \beta \frac{1}{d_i} \sum_{j \in N_i} y_j + \varepsilon_i \tag{1}$$

under the assumption $\mathbb{E}(\varepsilon_i | \mathbf{x}, \mathbf{G}) = 0$. With 1 characteristic, there are 4 parameters to estimate: the intercept $\alpha$, individual effect $\gamma$, contextual peer effect $\delta$ and endogenous peer effect $\beta$. With $k$ characteristics, there are $2k + 2$ parameters to estimate.

The strict exogeneity assumption $\mathbb{E}(\varepsilon_i | \mathbf{x}, \mathbf{G}) = 0$ means that characteristics $\mathbf{x}$ and the network $\mathbf{G}$ are both exogenous relative to outcome $\mathbf{y}$. In other words, the problem of correlated effects has been solved. We discuss below the various strategies used in the literature to address this central problem. We will show that this assumption can be relaxed. In particular, exogeneity of either some characteristic (random shocks) or of the network (random peers) can be sufficient to identify endogenous peer effects.

In some contexts, agents are partitioned in communities, such as classrooms or villages, with no connection between agents in different communities. The overall network is then partitioned in disconnected subnetworks and agents' outcomes may be affected by community fixed effects:

$$y_i = \alpha_C + \gamma x_i + \delta \frac{1}{d_i} \sum_{j \in N_i} x_j + \beta \frac{1}{d_i} \sum_{j \in N_i} y_j + \varepsilon_i \tag{2}$$

for agent $i$ in community $C$, under the assumption that $\mathbb{E}(\varepsilon_i | \boldsymbol{\alpha}, \mathbf{x}, \mathbf{G}) = 0$, where $\boldsymbol{\alpha}$ is the vector of community fixed effects $\alpha_C$.

Model (1) can be written in matrix notations:

$$\mathbf{y} = \alpha \mathbf{1} + \gamma \mathbf{x} + \delta \mathbf{G} \mathbf{x} + \beta \mathbf{G} \mathbf{y} + \boldsymbol{\varepsilon}$$

When the matrix $\mathbf{I} - \beta \mathbf{G}$ is invertible and no individual is isolated, this system of simultaneous linear equations is equivalent to the reduced-form equation expressing outcomes as function of structural parameters, the interaction matrix, and characteristics:

$$\mathbf{y} = \frac{\alpha}{1-\beta} \mathbf{1} + (\mathbf{I} - \beta \mathbf{G})^{-1} (\gamma \mathbf{I} + \delta \mathbf{G}) \mathbf{x} + (\mathbf{I} - \beta \mathbf{G})^{-1} \boldsymbol{\varepsilon}$$

In particular, $\mathbb{E}(\mathbf{y} | \mathbf{x}, \mathbf{G})$ is an affine function of $\mathbf{x}$. Model (1) is identified conditional on network $\mathbf{G}$ if the function $(\alpha, \beta, \gamma, \delta) \rightarrow (\frac{\alpha}{1-\beta} \mathbf{1}, (\mathbf{I} - \beta \mathbf{G})^{-1} (\gamma \mathbf{I} + \delta \mathbf{G}))$ is injective. In that case, the structural parameters $(\alpha, \beta, \gamma, \delta)$ can be uniquely recovered from the reduced-form relation between $\mathbb{E}(\mathbf{y} | \mathbf{x}, \mathbf{G})$

and $\mathbf{x}$. We characterized the relevant identification conditions in our previous work.

**Theorem 1** *(Bramoullé, Djebbari, and Fortin, 2009). Consider network* $\mathbf{G}$ *without isolated individuals and suppose that* $\delta + \gamma\beta \neq 0$*. Model (1) is identified if and only if* $\mathbf{I}$, $\mathbf{G}$ *and* $\mathbf{G}^2$ *are linearly independent. Model (2) is identified if* $\mathbf{I}$, $\mathbf{G}$, $\mathbf{G}^2$ *and* $\mathbf{G}^3$ *are linearly independent.*

To see where these conditions come from, consider $|\beta|$ small enough and develop the matrix inverse as an infinite series: $(\mathbf{I} - \beta\mathbf{G})^{-1} = \mathbf{I} + \beta\mathbf{G} + \beta^2\mathbf{G}^2 + \ldots$. In model (1) when no individual is isolated, we can express expected peers' outcomes as follows:

$$\mathbb{E}(\mathbf{Gy}|\mathbf{x}) = \frac{\alpha}{1-\beta}\mathbf{1} + \gamma\mathbf{Gx} + (\delta + \gamma\beta)(\mathbf{G}^2 + \beta\mathbf{G}^3 + \ldots)\mathbf{x}$$

When $\delta + \gamma\beta \neq 0$, peers' outcomes are affected by their characteristics $\gamma\mathbf{Gx}$, their peers' characteristics $(\delta + \gamma\beta)\mathbf{G}^2\mathbf{x}$, their peers' peers characteristics $(\delta + \gamma\beta)\beta\mathbf{G}^3\mathbf{x}$, and so on. Identification holds under intransitivity, when some peers of peers of an agent are not her peers. The variable $\mathbf{G}^2\mathbf{x}$ can then be used as an instrument for $\mathbf{Gy}$ in the estimation of Model (1). Other valid instruments can be built, some of which may be more robust to misspecification. These include the average characteristics of peers' peers who are not peers, other moments of these characteristics such as the sum or the variance, and the average characteristics of agents at distance 3 or higher in the network. Models (1) and (2) can also be estimated by maximum likelihood, under additional assumptions on the error terms, e.g. Lee, Liu, and Lin (2010).

**Broader validity of the identification strategy.** The main insight behind Theorem 1 is that, conditional on having addressed the problem of correlated effects, the characteristics of peers of peers who are not peers affect an individual's outcome only through their effect on peers' outcomes. This insight applies quite generally in network contexts. Let $\mathbf{x}_{N_i} = (x_j)_{j \in N_i}$ denote the vector of peers' characteristics, and similarly for $\mathbf{y}_{N_i}$. This identification strategy operates in any model of the form

$$y_i = \varphi(\mathbf{y}_{N_i}, x_i, \mathbf{x}_{N_i}, \boldsymbol{\beta}) + \psi(x_i, \mathbf{x}_{N_i}, \boldsymbol{\gamma}) + \varepsilon_i \tag{3}$$

where an agent's outcome is affected by her peers' outcomes in a way which could be non-linear, involve other moments than the average, and depend on her own and her peers' characteristics.[4]

[4] Under non-linearity, equations (3) define a fixed-point system in outcomes $\mathbf{y}$ which may have multiple or no solutions, yielding an incomplete econometric model. The researcher must first address this issue, for instance by specifying a selection mechanism, before analyzing identification and estimating the model (see de Paula, 2013)

The critical assumption is that an agent's outcome can only be affected *a priori* by the outcomes and characteristics of her peers. Since the characteristics of agents located at distance 2 or more in the network do not appear on the right-hand side of equation (3), they can be used to build valid instruments for peers' outcomes.

In the literature, researchers have considered many extensions of model (1) and (2) and have generalized the identification results of Bramoullé, Djebbari, and Fortin (2009), maintaining the assumption that the problem of correlated effects has been solved. In particular, the interaction matrix may not be row-normalized, peer effects could be heterogeneous, outcomes could be discrete, and there could be multiple outcomes. *First, the matrix of interactions may not be row-normalized.* This notably happens in the presence of isolated individuals and in a linear-in-sums formulation where individual outcome is affected by the sum of her peers' characteristics and outcomes.[5] An implication of relaxing row-normalization is that intercepts in the reduced-form equations now vary across individuals depending on their positions in the network. The vector of intercepts is equal to $\alpha(\mathbf{I}-\beta\mathbf{G})^{-1}\mathbf{1} = \alpha(\mathbf{1}+\beta\mathbf{b})$, where $\mathbf{b}$ is the vector of Katz-Bonacich centralities (see Bonacich, 1987; Calvó-Armengol, Patacchini, and Zenou, 2009). These variations can then be exploited for identification and estimation (see Liu and Lee, 2010).[6]

For instance, in a linear-in-means model with undirected links and isolated individuals, the intercept is $\alpha$ for isolated individuals and $\alpha/(1-\beta)$ for non-isolated individuals. The endogenous peer effect can then, in principle, be identified by contrasting intercepts in the reduced-form regressions of isolated and non-isolated agents. This identification strategy may raise concerns in practice, however. Apparently isolated individuals may, in fact, be socially connected if the network is measured with error (see Section 4). And truly isolated individuals may differ in systematic ways, generating another source of intercept differences. The treatment of apparently isolated individuals in empirical studies generally requires empirical care, given their quantitative importance in many datasets. For instance, apparently isolated individuals represent 23% of the Add Health data set used in Dieye and Fortin (2017).

*Second, peer effects may be heterogeneous.* For instance, Beugnot et al. (2019) consider a variant of model (1) where men and women are subject to different peer effects. Individuals could also be subject to different effects from male peers and from female peers, potentially leading to four kinds of endogenous peer effects (see Arduini, Patacchini, and Rainone, 2019a,b; Bramoullé, 2013). Masten (2018) incorporates heterogeneity in peer effect analysis by assuming that endogenous peer

[5] Individual $i$ is isolated if $g_{ij} = 0$ for any $j$. In a linear-in-sums formulation, the interaction matrix $\mathbf{G}$ is the adjacency matrix of the network: $g_{ij} = 1$ if $j \in N_i$ and 0 otherwise.

[6] Individual outcome could also depend on both the mean and the sum of peers' outcomes, see Liu, Patacchini, and Zenou (2014).

effect coefficients are random in a linear-in-means model. He notably shows that the marginal distributions of these random endogenous peer effects can be point-identified if there is no contextual peer effect with respect to an exogenous characteristic. Can the network-based identification strategy be adapted to a random coefficient framework? Whether Masten (2018)'s results extend to a network model with both endogenous and contextual peer effects is, at this stage, an open question.

*Third, outcome could be discrete.* Smoking, for instance, is often analyzed as a binary outcome. Discreteness deeply affects the econometric analysis (see Kline and Tamer, 2019). Models of endogenous peer effects with discrete outcomes generally display non-linearities and multiple equilibra under group interactions (see Brock and Durlauf, 2001). While non-linearities help identification, multiplicity complicates the analysis. Lee, Li, and Lin (2014) extend Brock and Durlauf (2001)'s framework and analysis to networks.[7] They consider a network game of incomplete information and show that the model has a unique equilibrium if the endogenous peer effects parameter is small enough in magnitude. They apply their framework to analyze peer effects in smoking on Add Health data. They find evidence that both contextual and endogenous peer effects matter. Identification in their analysis exploits both non-linearities and the network structure.

*Fourth, multiple outcomes may be affected by peer effects.* For instance, friends of a high school student may influence both his alcohol and tobacco consumption. Moreover, alcohol and tobacco may be complements in the student's preferences (Tauchmann et al., 2013) and peer effects may also involve cross effects. An individual could be more likely to smoke when her friends consume more alcohol. Cohen-Cole, Liu, and Zenou (2018) develop a linear-in-means model of peer effects with two outcomes. One outcome depends on community fixed effects, on the other outcome ("self-simultaneity"), on the average of this outcome among peers ("within-activity endogenous peer effect"), on the average of the other outcome among peers ("cross-activity endogenous peer effect") and on individual and contextual peer effects. Authors extend the identification analysis of Bramoullé, Djebbari, and Fortin (2009) to their setting. They show that the model is generally *not* identified because of self-simultaneity. Identification holds, however, with classical exclusion restrictions and if the matrices $\mathbf{I}, \mathbf{G}, \mathbf{G}^2, \mathbf{G}^3$ and $\mathbf{G}^4$ are linearly independent.

**Groups and size effects.** Suppose that agents interact in groups and that mean peer characteristics and outcomes are computed over every agent in the group. This formulation is equivalent to the linear-in-expectations model considered in Manski (1993), when analyzing identification without further assumptions on the error terms. Under such inclusive averaging, $\mathbf{G}^2 = \mathbf{G}$ and model (1) is not identified. The researcher can recover the reduced-form impact of average peer characteristics on individual outcome, but cannot disentangle endogenous and contextual peer effects. This is the

[7] See Li and Zhao (2016) for an alternative approach based on partial identification.

essence of the reflection problem discussed in Manski (1993). And this reduced-form impact is, of course, not identified in the presence of group fixed effects, undistinguishable from common shifters.

Surprisingly, however, identification under group interactions critically relies on the way the average is computed. Assume instead, as usually done in empirical studies, that mean peer characteristics are computed over everyone *else* in the group. Under exclusive averaging, models (1) and (2) are identified if there is sufficient variation in group sizes. This result was first discovered by Lee (2007) and the corresponding model was first estimated on data by Boucher et al. (2014), see also Davezies, d'Haultfoeuille, and Fougère (2009). This result appears, at first sight, to be quite extraordinary. How could peer effects be identified at all, let alone in their contextual and endogenous components, when agents interact with everyone in their group and outcomes can be affected by common correlated shocks? Unlike with a network structure, identification is not immediately transparent here and this might explain a relative lack of exploration of this identification strategy in recent studies.

As it turns out, identification in that case exploits effects of group size induced by the linear-in-means formulation. Under exclusive averaging, higher ability agents have lower ability peers and vice versa. This mechanical negative correlation tends to reduce the dispersion in outcomes under positive peer effects, and this reduction is larger in smaller groups.[8] Formally if agent $i$ belongs to group $C$ of size $n_C$, model (2) reduces to:

$$y_i - \bar{y}_C = \frac{\gamma - \frac{\delta}{n_C - 1}}{1 + \frac{\beta}{n_C - 1}}(x_i - \bar{x}_C) + v_i$$

where $\bar{y}_C = \frac{1}{n_C}\sum_{j \in C} y_j$ and similarly for $\bar{x}_C$. A critical identifying assumption is then that the structural parameters $\beta, \gamma$ and $\delta$ do not depend on group size.[9] Empirical applications of this model then fall, in fact, within the broader study of size effects (e.g. Angrist and Lavy, 1999; Hoxby, 2000; Krueger, 2003).[10] Note that model (2) is a particular case of the following model:

$$y_i = \alpha_C + \gamma(n_C)x_i + v_i$$

where $\alpha_C$ is a group fixed effect and the impact of characteristic $x$ on outcome $y$, $\gamma(n_C)$, can depend in an arbitrary way on group size $n_C$. The function $\gamma(\cdot)$ can be estimated non-parametrically if the

[8] A similar mechanical correlation underlies the exclusion bias analyzed by Caeyers and Fafchamps (2019).

[9] Identification of course tends to be weak when groups have large sizes although group size variation helps (see Lee, 2007; Boucher et al., 2014).

[10] In Graham (2008)'s conditional variance restrictions model (see our discussion below), an important identifying assumption is also that the structural parameters are not affected by group size.

researcher has enough observations of groups of every size. Linear-in-means peer effects provide a key candidate explanation for the presence of size effects in characteristics' impacts, and induce testable restrictions on the function $\gamma(\cdot)$. If they are the only source of size effects, $\gamma(\cdot)$ is monotone in $n_C$ and its shape depends on contextual and endogenous peer effects. When $\beta = 0$, $\gamma(\cdot)$ is linear in $1/(n_C - 1)$ with slope $-\delta$. By contrast, $\gamma(\cdot)$ is strictly concave or strictly convex in $1/(n_C - 1)$ when $\beta \neq 0$ and its curvature is more pronounced when endogenous peer effects are larger in magnitude.

**Identification based on assumptions on the error terms.** The benchmark model (1) does not impose any restriction on conditional covariances between the error terms, and more generally, on their distributions. Introducing such restrictions of course helps identification.[11] In particular since $\boldsymbol{\nu} = (\mathbf{I} - \beta\mathbf{G})^{-1}\boldsymbol{\varepsilon}$, the endogenous peer effect $\beta$ can potentially be identified from features of $\boldsymbol{\nu}$ induced by assumptions on $\boldsymbol{\varepsilon}$. For instance when agents interact in pairs, Moffitt (2001) shows that model (1) is identified if the error terms are uncorrelated, $\mathbb{E}(\varepsilon_i\varepsilon_j|\mathbf{x}, \mathbf{G}) = 0, \forall i \neq j$. Liu (2017) extends this idea. He assumes that agents interact in groups of the same size and that errors are uncorrelated. He shows that model (1) is identified and can be consistently estimated using a root estimator (see Ord, 1975). The assumption of uncorrelated errors is strong and is more likely to be satisfied in experimental contexts, when individuals are randomly assigned to groups by the experimentalist, e.g. Fortin, Lacroix, and Villeval (2007). As pointed out by Moffitt (2001), however, the error terms are likely to be correlated in most applications, due for instance to endogenous group formation.

Graham (2008) shows that peer effects can be identified from conditional covariance restrictions, even in the absence of exogenous variations in observed characteristics among agents. He considers a linear-in-means model of peer effects in a group context including contextual peer effects relative to the error term, but no endogenous peer effect, and random effects at the group level, under a number of assumptions. His setup notably applies to education contexts under double randomization of students and teachers to classrooms. He shows that the magnitude of the peer effect can be identified by contrasting outcome variance between and within groups across groups of different sizes. Rose (2017) extends Graham (2008)'s ideas to network interactions. He considers a model with contextual and endogenous peer effects and unrestricted group fixed effects, but no impact of observables. Extending the arguments in Bramoullé, Djebbari, and Fortin (2009), he shows how the structure of the network can be used to identify contextual and endogenous peer effects. A key argument is that outcome covariance between peers of peers who are not directly connected only depends on the endogenous peer effect, once unobserved group heterogeneity is accounted for.

[11] There exists an important literature on identification in simultaneous equation models with covariance restrictions (e.g. Hausman, Newey, and Taylor, 1987).

**Knife-edge identification failure.** In an influential overview of the literature, Angrist (2014) presents sharp criticisms of various econometric approaches and empirical studies of peer effects. In Section 6, in particular, he criticizes the approach developed in Bramoullé, Djebbari, and Fortin (2009). He builds a specific case of model (1) where apparent peer effects "reflect tautological relationships and quotidian correlation in unobservables, in a world otherwise characterized by social indifference"(Angrist, 2014, p. 10). We show next that his example simply illustrates a well-known situation of identification failure.

To see why, reproduce equation (30) in Angrist (2014) using our notations:

$$y_i = \alpha + \beta y_{i-1} + \gamma x_i - \gamma\beta x_{i-1} + \varepsilon_i$$

This is, indeed, a particular case of model (1). Note, however, that Angrist imposes $\delta + \gamma\beta = 0$, which violates the identification condition spelled out in Proposition 1 in Bramoullé, Djebbari, and Fortin (2009). More generally when $\delta + \gamma\beta = 0$, model (1) is equivalent to:

$$y_i = \frac{\alpha}{1-\beta} + \gamma x_i + \nu_i$$

In this knife-edge situation, contextual and endogenous peer effects exactly cancel out and models with or without peer effects are observationally equivalent for any network without isolated agents. In other words, Angrist (2014)'s example simply confirms that the condition $\delta + \gamma\beta \neq 0$ is necessary for identification. This identification failure is non-generic and disappears under sign restrictions, in the presence of isolated individuals or under restrictions on the error terms. It cannot happen, for instance, when $\gamma$ and $\delta$ have the same sign, $\beta > 0$ and $\gamma \neq 0$. In our view, the arguments developed by Angrist (2014) thus do not invalidate the use of the network structure to help identify peer effects.

By contrast, Angrist (2014) highlights some important issues in other Sections. He emphasizes, in particular, that measurement error on observed characteristics can artificially inflate peer effect estimates (see his Table 3 and Moffitt (2001) p.23-24 for an early discussion). To see why, note that averaging in general reduces noise. Average peer characteristics are then better measured than individual characteristics. When true characteristics are correlated across peers, average peer characteristics can act as a proxy for the true individual characteristic. Apparent estimates of peer effects may then simply be picking up part of the individual effect. This *expansion bias* operates in an opposite way to the standard attenuation bias and may well be a first-order empirical problem in many contexts. The literature is still missing a comprehensive analysis of measurement error on observed characteristics and outcomes in regressions of peer effects in networks based on models (1) and (2).

**Mechanisms and microfoundations.** A well-identified estimation of peer effects only constitutes a first step in the analysis of social interactions. Peer effects can have different causes including complementarities, conformism, social status, social learning and informal risk sharing. Any of these motives could, potentially, generate the kind of peer effects considered in models (1) and (2). The distinction between contextual and endogenous peer effects, while informative, is generally too coarse to identify precise causes. Different mechanisms have different welfare implications and could further interact in complex ways. Making progress on this issue is an important next step for researchers. Identifying the precise causes behind peer effects will require econometric, experimental and empirical creativity.

Relatedly, researchers have proposed various microfoundations of linear models of peer effects. A natural idea here, at least when outcomes represent actual choices, is to interpret the econometric equation describing individual outcome $y_i$ as a function of others' outcomes $y_{-i}$ as the best response of a non-cooperative game. This connects the estimation of peer effects to the econometrics of games. A fundamental problem with this approach, however, is that different preferences are generally compatible with a given best response. This is a main reason why the literature on revealed preferences in strategic contexts is much less developed than in the usual consumer framework.

The problem of preference non-identification holds even when making strong restrictions, for instance when focusing on quadratic utility functions, see Blume et al. (2015). To illustrate, denote by $\bar{x}_i = \frac{1}{d_i}\sum_{j\in N_i} x_j$, and similarly for $\bar{y}_i$, and by $\alpha_i = \alpha + \gamma x_i + \delta\bar{x}_i + \varepsilon_i$ the part of the right-hand side of model (1) which does not depend on peers' outcomes. Then model (1) describes the best response of a game with quadratic utilities if and only if there exists real numbers $A_i > 0$ and quadratic functions $v_i : \mathbb{R}^{n-1} \to \mathbb{R}$ such that

$$u_i(y_i, \mathbf{y}_{-i}) = A_i(\alpha_i y_i - \frac{1}{2}y_i^2 + \beta\bar{y}_i y_i) + v_i(\mathbf{y}_{-i}) \tag{4}$$

An empirical researcher exploiting data on best responses cannot recover the functions $v_i(\cdot)$. While these functions have no effect on individual actions, they critically affect the externality patterns and the welfare properties of these games.

Underlying preferences also affect counterfactual reasoning. This concerns, in particular, the existence of a social multiplier. Applied researchers estimating variants of models (1) and (2) often claim that endogenous peer effects give rise to a social multiplier. This widespread belief turns out to be incorrect, as shown by Boucher and Fortin (2016). Standard reasoning runs as follows. Suppose that, before peer effects operate, individuals are subject to a shock which changes outcomes by amount $\Delta$. This initial shock is then transformed through endogenous peer effects, yielding an outcome change of $\Delta/(1-\beta)$ for any network without isolated agents. This reasoning is indeed valid, for instance, for utilities with complementarities $u_i(y_i, \mathbf{y}_{-i}) = \alpha_i y_i - \frac{1}{2}y_i^2 + \beta\bar{y}_i y_i$.

This reasoning may not apply, however, for other utility functions compatible with the same econometric model. The problem lies in the implicit assumption that the initial shock directly affects the intercept of the econometric equation. To see why, consider utilities for conformity $u_i(y_i, \mathbf{y}_{-i}) = \alpha_i' y_i - \frac{1}{2}y_i^2 - \frac{1}{2}\lambda(y_i - \bar{y}_i)^2$ where $\alpha_i'$ depends on individual and peers' characteristics and $\lambda$ captures preference for conformity. The corresponding econometric equation is now:

$$y_i = \frac{\alpha_i'}{1+\lambda} + \frac{\lambda}{1+\lambda}\bar{y}_i$$

which is a version of model (1) with $\alpha_i = \alpha_i'/(1+\lambda)$ and $\beta = \lambda/(1+\lambda)$.[12] The initial shock increases $\alpha_i'$ by amount $\Delta$, leading to an increase in the intercept of model (1) by amount $\Delta/(1+\lambda)$. Once endogenous peer effects operate, this becomes $\Delta$ again. Thus, common shocks on individuals are passed on outcomes and generate no multiplier. This is due to the conformity motive: if all actions change by a common amount, $|y_i - \bar{y}_i|$ is unaffected. Since agents care about the absolute value of the difference between their action and their peers' average action, common shocks do not generate indirect effects. Under conformity preferences, endogenous peers effects do not give rise to a social multiplier.

This example teaches us some important lessons. Applied researchers should generally refrain from making counterfactual predictions based on specifications like models (1) and (2). Counterfactual predictions require a precise, structural understanding of the causes behind peer effects. This concerns many issues ranging from social multiplier computations to optimal classroom composition. Indeed, this is likely a main reason behind the failure of the social engineering experiment implemented in Carrell, Sacerdote, and West (2013).

# 3 Correlated effects and networks

Researchers who wish to provide credible causal estimates of peer effects and to exploit the identification possibilities generated by interaction networks must address, somehow, the problem of correlated effects. Individual unobserved characteristics may be correlated with peers' characteristics and the interaction matrix, because of common shocks or network endogeneity (e.g. an individual and her peers tend to have similar preferences). In the literature, researchers have developed at least four broad strategies to address this critical issue: random peers, random shocks, structural endogeneity and panel data. We discuss each of these identification strategies in turn and how they can be com-

[12] The utility for conformity is thus a case of quadratic utility (4) leading to the same best responses, with $A_i = 1+\lambda$ and $v_i(\mathbf{y}_{-i}) = -\frac{1}{2}\lambda\bar{y}_i^2$.

bined with the identification of contextual and endogenous peer effects via the network structure. We review relevant applied and econometric studies.

## 3.1 Random peers

**Natural and artificial experiments.** To estimate causal impacts of peers, researchers have been studying contexts where peers are randomly allocated, through natural or artificial experiments. Many studies consider random peers with a group structure. Sacerdote (2001) looks at roommates in pairs, triples or quads and dormmates among students entering Dartmouth College. Carrell, Sacerdote, and West (2013) consider squadrons of freshmen at the Air Force academy. Falk and Ichino (2006) randomly match workers in pairs in the lab. Note that researchers have generally not tried to identify contextual and endogenous peer effects from data on random groups of peers. Instead, they focus on one or the other effect.

More recently, researchers have analyzed network data where peers are formed at random. De Giorgi, Pellizzari, and Redaelli (2010) look at the choice of major among Bocconi undergraduates. Students initially attend lectures in randomly assigned classes, for nine compulsory courses. This allows researchers to build interaction matrices from common class assignments. In their preferred specification, they assume that two students are peers if they attended at least four of the seven common compulsory courses in the same classes. In this context, the network of interactions itself is random. In other contexts, the network is predetermined and individuals are randomly positioned in it, see Beugnot et al. (2019) in the lab and Mas and Moretti (2009) in the field.

**Random peers and omitted variables.** With random peers, what can be causally identified, precisely? We believe that the literature has been, overall, quite unclear on this central question. Start with a simple case: assume that individuals interact in random pairs. A researcher has data on past smoking behavior and is interested in assessing the impact of peer smoking on grades. Contrasting grades of individuals paired, by chance, with a smoker to grades of individuals paired with a non-smoker allows the researcher to identify the causal impact of being paired with a smoker relative to a non-smoker on grades. This causal statement lies at the heart of the interpretation of results in most studies based on random peer groups.

We should be careful about interpretation, however. The researcher here does *not* estimate the causal impact of peers' smoking behavior. This is due to omitted variables: even if being paired, by chance, with a smoker leads to a drop in grades, this may have nothing to do with peer smoking. Perhaps smokers, on average, have lower ability or drink more and peers' ability or drinking behavior is what really drives peer effects. Random allocation of peers does not solve this problem. To estimate the causal impact of peer smoking on grade, an exogenous shock on peer smoking is needed. In our view, this important issue limits the relevance of random peers to identify and estimate contextual peer effects.

More generally, potential problems of omitted variables pervade empirical research based on random matching between agents. To illustrate, consider an interesting and important analysis by Breza and Chandrasekhar (2019). The authors conduct an experiment to study whether individuals in rural Indian villages save more when information about the progress toward their self-set savings goal is shared with another village member, a monitor. In some treatments, the monitor is randomly assigned and authors find that savings increase when the monitor has a more central position in the village's social network. Authors make use of innovative high-dimensional econometric techniques to verify that this apparent impact of monitor's centrality is not caused by correlated observables. It could, however, potentially be caused by correlated unobservables. *Even with random peers, then, researchers who wish to identify causal impacts of peers' characteristics face a classical problem of omitted variables.* Absent other sources of exogenous variation, they must address this problem in classical ways: they must ascertain likely confounders, depending on the context, and control for relevant variables and proxies.

**Identification with random network peers.** By contrast, we now show that under network interactions, random peers allow researchers to identify endogenous peer effects. The key property is that with random peers an agent's observed and unobserved characteristics are uncorrelated with her peers' observed and unobserved characteristics. To illustrate, consider, again, the relation between smoking, peers and academic achievement. Suppose that any two students are linked with the same probability. The proportion of smokers among peers of peers who are not peers is uncorrelated with individual observed and unobserved characteristics. This proportion thus affects individual achievement only through its effect on peers' outcomes and hence provides a valid instrument for peers' outcomes in the main regression. By contrast, as with groups, estimates of contextual peer effects may not capture the causal impact of the proportion of smoking peers $\bar{x}_i$. Rather, they capture the causal impact of being connected, by chance, to peers with average smoking rate $\bar{x}_i$ and this impact may have nothing to do with smoking because of omitted variables.

Formally, consider the following variant of model (1)

$$y_i = \alpha + \gamma_x x_i + \gamma_u u_i + \delta_x \bar{x}_i + \delta_u \bar{u}_i + \beta \bar{y}_i + \varepsilon_i \tag{5}$$

where $\mathbb{E}(\varepsilon_i|\mathbf{x}, \mathbf{u}, \mathbf{G}) = 0$ and $u_i$ is unobserved and potentially correlated with $x_i$, $u_i = a + bx_i + \eta_i$ with $\mathbb{E}(\eta_i|x_i) = 0$. Denote by $\bar{\bar{x}}_i$ the average value of $x$ among peers of peers of $i$ who are not peers of $i$. With random peers, observed characteristics of agents two-step away in the network are uncorrelated with unobserved characteristics of the individual and her peers:

$$cov(\bar{\bar{x}}_i, u_i) = cov(\bar{\bar{x}}_i, \bar{u}_i) = 0$$

This implies that $\bar{\bar{x}}_i$ provides a valid instrument for $\bar{y}_i$ in model (5), and endogenous peer effects are identified. To see what happens for contextual peer effects, rewrite model (5) in terms of observables. With random peers, $\mathbb{E}(\eta_i|x_i, \bar{x}_i) = 0$ and $\bar{u}_i = a + b\bar{x}_i + \bar{\eta}_i$. This yields:

$$y_i = \alpha + (\gamma_x + \gamma_u)a + (\gamma_x + \gamma_u b)x_i + (\delta_x + \delta_u b)\bar{x}_i + \beta\bar{y}_i + \nu_i$$

with $\mathbb{E}(\nu_i|x_i, \bar{x}_i) = 0$. We recognize the classical formula for omitted variables. The *apparent* contextual peer effect $\delta_x + \delta_u b$ aggregates the causal impact of peers' observed characteristics $\delta_x$ and the causal impact of peers' unobserved characteristics $\delta_u$ times the conditional correlation between the unobserved and the observed characteristics $b$. In particular, $\delta_x + \delta_u b \neq 0$ implies that $\delta_x \neq 0$ or $\delta_u \neq 0$. Detecting apparent contextual peer effects indicates that some average peer characteristics, observed or unobserved, have a causal impact on individual outcomes.

These conclusions also hold under group interactions. With random peer groups and if model (5) is the correct model, researchers can empirically recover apparent contextual peer effects $\delta_x + \delta_u b$ and endogenous peer effects $\beta$ from group size variation. Identification then relies on size effects in the reduced-form impact of individual and average group characteristics, as discussed in Section 2. To our knowledge, no study has tried to estimate models (1) or (2) on data with random peer groups. It would be interesting to reanalyze the data from key studies on random peer groups to estimate variants of models (1) and (2). This would help advance our understanding of the validity, performance and robustness of the identification strategy based on group size variation.

**Observational data.** In the literature, many studies of peer effects based on observational data rely on quasi-random peers to address the problem of correlated effects. In the best cases, peer assignment is as good as random conditional on observables and our previous arguments apply. The endogenous peer effect is identified under network interactions and estimates of contextual peer effects may not capture the causal impact of peer characteristics. For instance, a standard identification strategy in an education context and under group interactions is to exploit quasi-random variations in observable characteristics across cohorts within a school (see Hoxby, 2000). The identifying assumption here is that, conditional on school fixed effects, variations in cohort characteristics such as the share of female students are as good as random. And indeed, many studies analyzing gender effects in education rely on this identification strategy, (e.g. Feld and Zolitz, 2017; Hoekstra, Mouganie, and Wang, 2018; Cools, Fernández, and Patacchini, 2019). Because of omitted variables, however, estimates of the impact of the share of female peers on individual outcomes may not capture the causal impact of peers' gender.

Studies of peer effects in networks which do not rely on some explicit exogenous shock (see the next Section) also address the problem of correlated effects, implicitly or explicitly, by assuming that the network is as good as random conditional on the observables. This notably concerns most papers analyzing peer effects in networks using datasets from the National Longitudinal Study of Adolescent to Adult Health (Add Health) (e.g. Trogdon, Nonnemaker, and Pais, 2008; Calvó-Armengol, Patacchini, and Zenou, 2009; Liu, Patacchini, and Rainone, 2017). Are self-reported friendships in Add Health conditionally quasi-random? Given the importance of this dataset in the analysis of peer effects, we believe that this question deserves a separate, focused investigation.

## 3.2 Random shocks

When peers are not random, researchers have been trying to identify peer effects using other sources of exogenous variations. This connects the analysis of peer effects to the large literature on randomized interventions. In this Section, we first discuss how researchers can combine a randomized treatment with the network structure to identify contextual and endogenous peer effects in a linear-in-means framework. Identification holds even with endogenous peers as long as the network is not affected by the treatment. The linear-in-means model imposes strong restrictions, however. We then discuss another strand of the literature, closer to the statistical tradition prevalent in the analysis of treatment effects, which tries to avoid parametric restrictions. Researchers in that literature analyze what can be learned on spillovers from randomized treatments and how this depends on assumptions made on the structure of these spillovers.

**Random shocks and linear-in-means peer effects.** As shown by Dieye, Djebbari, and Barrera-Osorio (2017), treatment randomization allows researchers to address the problem of correlated effects and, within a linear-in-means framework, to identify the causal impacts of the treatment and of peers' treatments and peers' outcomes even when the network is endogenous. To see why, consider a variant of model (5) with unobserved characteristics and under the assumption that there is no isolated individual. Assume that each individual is treated with probability $q$, that individual treatments are independent and that the network of interactions is not affected by the treatment. Let $t_i = 1$ if $i$ is treated and $t_i = 0$ otherwise.

$$y_i = \alpha + \gamma_t t_i + \gamma_u u_i + \delta_t \bar{t}_i + \delta_u \bar{u}_i + \beta \bar{y}_i + e_i$$

with $\mathbb{E}(e_i|\mathbf{t},\mathbf{u},\mathbf{G}) = 0$. The network may be endogenous: $\mathbb{E}(u_i|\mathbf{G}) \neq 0$. Here $\bar{t}_i$ represents the share of treated peers. Incorporate individual and peers' unobserved characteristics in the error term, $\varepsilon_i = \gamma_u u_i + \delta_u \bar{u}_i + e_i$:

$$y_i = \alpha + \gamma_t t_i + \delta_t \bar{t}_i + \beta \bar{y}_i + \varepsilon_i$$

Randomization implies that $cov(t_i, u_j) = 0$ for every pair $i$, $j$. Because the interaction matrix is row-normalized, applying the law of iterated expectations leads to[13]

$$cov(t_i, u_i) = cov(\bar{t}_i, u_i) = cov(t_i, \bar{u}_i) = cov(\bar{t}_i, \bar{u}_i) = 0$$

which means that individual treatment and the share of treated peers are uncorrelated with the error term: $cov(t_i, \varepsilon_i) = cov(\bar{t}_i, \varepsilon_i) = 0$.[14] More generally, for any power $k$ of the interaction matrix, $cov((\mathbf{G}^k\mathbf{t})_i, \varepsilon_i) = 0$. In particular, the weighted share of treated peers of peers, $(\mathbf{G}^2\mathbf{t})_i$ is uncorrelated with the error term and hence provides a valid instrument for peers' outcomes. Therefore, the three parameters $\gamma_t$, $\delta_t$ and $\beta$ are causally identified with random shocks in a linear-in-means framework and even when the network is endogenous.

This conclusion also holds in a model with individual and contextual peer effects from predetermined observed characteristics. These effects can be included in regressions although their estimates do not generally have a causal interpretation. In addition, predetermined characteristics of peers of peers who are not peers may *not* provide valid instruments for peers' outcomes when the network is endogenous. Therefore, only instruments built from the treatments of indirectly connected agents should be included in instrumental regressions of peer effects exploiting the network structure.

Interestingly, the identification of peer effects may not hold when the network is endogenous and the interaction matrix is not row-normalized. For instance, in a linear-in-sums model and if $u_i$ is positively correlated with degree, then $cov(u_i, \sum_j g_{ij} t_j) > 0$. Agents with more friends tend to have more treated friends and the estimate of the impact of the *number* of treated friends on outcomes is biased, even with a randomized treatment. This shows that *the identification of peer effects with randomized treatments depends on the specific model of peer effects considered*. Peer effects may be identified in one model (linear-in-means) but not in another (linear-in-sums).

Because of peer effects, the impact of the treatment cannot be estimated by the naive difference in average outcomes between treated and untreated individuals. Dieye, Djebbari, and Barrera-Osorio

[13] For instance, note that $\mathbb{E}(u_i\bar{t}_i) = \mathbb{E}(\sum_j g_{ij} u_i t_j) = \mathbb{E}_{\mathbf{G}}(\mathbb{E}(\sum_j g_{ij} u_i t_j|\mathbf{G})) = \mathbb{E}_{\mathbf{G}}(\sum_j g_{ij} \mathbb{E}(u_i|\mathbf{G})\mathbb{E}(t_j)) = q\mathbb{E}_{\mathbf{G}}(\mathbb{E}(u_i|\mathbf{G})) = \mathbb{E}(u_i)\mathbb{E}(\bar{t}_i)$ where the second equality comes from the law of iterated expectations, the third equality from the fact that $\mathbf{t} \perp\!\!\!\perp \mathbf{u}, \mathbf{G}$, and the fourth equality from row-normalization.

[14] By contrast, we can have $\mathbb{E}(u_i|\bar{t}_i) \neq 0$ when unobserved characteristics are correlated with degree.

(2017) propose a measure of treatment effect, which can be decomposed into a direct effect and indirect effects due to contextual peer effects and to changes in outcomes mediated by endogenous peer effects. They also compute the difference in expected outcomes between treated and untreated individuals as a function of the parameters of the model, see their Proposition 2. This provides a testable prediction which can be used to validate the model, complementing overidentification tests.

Arduini, Patacchini, and Rainone (2019a) analyze the impact of a randomized treatment under group interactions and in a linear-in-means framework with heterogeneous peer effects. An important limitation of their framework, however, is that they assume that there is no contextual peer effect associated with the treatment. An individual is thus not directly affected by her peers' treatments and this assumption greatly facilitates identification.

Comola and Prina (2019) analyze a situation where the network is also affected by the randomized intervention. They consider a linear-in-means model with two periods and two networks: before and after the intervention. Outcome in the first period depends on individual fixed effects and average outcomes among old peers. Outcome in the second period depends on individual fixed effects, individual treatment, the shares of treated agents among old and new peers and average outcomes among old and new peers. The authors provide identification conditions, and associated instrumental variable strategies, extending those of Bramoullé, Djebbari, and Fortin (2009). They assume that the networks before and after intervention are conditionally exogenous. This condition may fail to hold, however, if some time-varying unobservable affects both network change and outcome. They propose a measure of treatment impact, which can be decomposed into a direct effect and indirect effects coming both from the treatment of baseline peers and from the change in the network. They use their model to estimate the impact of an intervention in rural Nepal providing randomized access to savings accounts. Their results suggest that neglecting network changes leads to underestimate the impact of the intervention. This paper contributes to an emerging literature analyzing the impact of interventions on social networks (see Banerjee et al., 2018).

**Observational studies.** In empirical studies of peer effects in networks based on observational data, researchers have exploited naturally occurring shocks. In the best cases, these shocks are as good as conditionally random and hence allow researchers to identify peer effects. De Giorgi, Frederiksen, and Pistaferri (2019) study peer effects in consumption based on administrative panel data on households in Denmark. They build a measure of household consumption based on tax records and assets and consider a network of coworkers. They assume that an individual is affected by people who work in the same plant and have similar education and occupation. An important feature of their framework is that consumption is shared among spouses and defined at the household level while working relationships are individual based. They consider a linear-in-means model with two kinds of endogenous peer effects: log household consumption depends on household fixed effects, average log consumption of the husband's coworkers, average log consumption of the wife's coworkers, and individual and contextual peer effects.[15] To address endogeneity issues, they account for time-invariant household characteristics and exploit firm-level shocks. In particular, they instrument the first-difference of the average log consumption of the husband's and the wife's coworkers by firm-specific variables of peers at distance 3: the coworkers of the spouses of the coworkers. Combining professional and marital connections helps ensure that intransitivity indeed holds: individuals have coworkers, these coworkers have spouses and these spouses interact with their own coworkers in another workplace. This approach is valid if the firm-specific variables are indeed conditionally exogenous. The authors find evidence of positive endogenous peer effects in consumption and further explore the mechanisms at work.

Nicoletti, Salvanes, and Tominey (2018) analyze peer effects on mothers' working hours in the year after childbirth based on administrative data from Norway. They consider a linear-in-means model of peer effects with two kinds of peers: family peers (sisters and female cousins who gave birth before the focal mother) and geographic neighbors (with similar education and who also gave birth before the focal mother). In their main regression, they instrument the working hours of family peers by the working hours of the neighbors of family peers controlling for the working hours of neighbors. Past labor supply of family peers' neighbors thus constitutes a kind of shock on family peers' labor supply. If this shock is conditionally exogenous, it can be used to identify the effect of family peers' working hours on mothers' working hours. They find evidence of positive family peer effects. They do not estimate a full-fledged extension of model (1), however, including both endogenous and contextual peer effects for the two kinds of peers.

**Randomized treatments and spillovers.** Linear-in-means peer effects are a particular case of situations where individual outcome is affected by others' treatments. Historically, such spillovers were considered a nuisance in the literature on treatment effects. With spillovers, the causal impact of a randomized treatment *cannot* be estimated by simply computing the difference between the average outcome among treated and among untreated individuals.[16] In light of mounting evidence that these spillovers are widespread and have a critical impact on the evaluation and design of public policies, the literature on treatment effects recently changed its point of view (see Miguel and Kre-

[15]The model is a variant of model (8) introduced in Section 3.4 on panel data, with no contextual peer effects associated with time-invariant unobserved characteristics.

[16]A common assumption in the literature on treatment effects is the Stable Unit Treatment Value Assumption (SUTVA), which rules out the possibility that potential outcomes may depend on the treatment of peers (see Rubin (1978)). The early negative view of spillovers is reflected in the use of expressions like "interference" and "contamination of the control group" to denote such situations.

mer, 2004; Kremer and Miguel, 2007; Crépon et al., 2013). Researchers are now trying to estimate these spillovers and to better understand their determinants, their implications and the experimental designs and assumptions under which they can be identified. In general, an individual's potential outcome may depend on the full vector of potential treatments and *no parameter of interest is identified without further assumptions and restrictions* (see Manski, 2013). One possible solution is to assume that agents are organized in groups and that spillovers take place within but not between groups, (e.g. Hudgens and Halloran, 2008; Vazquez-Bare, 2017). Spillovers may then be identified by comparing outcomes of untreated individuals in treated versus untreated groups, (e.g. Duflo and Saez, 2002; Angelucci and Giorgi, 2009).

This assumption generally does not hold, however, when agents interact through a network. In an emerging and fast-growing literature, researchers analyze detection, identification and estimation of treatment spillovers under network interactions (see Aronow and Samii, 2017; Athey, Eckles, and Imbens, 2018; Manski, 2013). Given information on the network of interactions, they explore the implications of various assumptions such as "no second and higher order effects" (treatment of agents at network distance 2 or more has no impact) and "no peer effect heterogeneity" (the number of treated peers may matter but not their identity). Athey, Eckles, and Imbens (2018) notably build exact randomization tests for different hypothesis on the presence and structure of spillovers. How these studies address problems of correlated effects and network endogeneity is quite unclear, however. As shown above, the identification of peer effects in networks with a randomized treatment depends on the specific model of peer effects. When the network is endogenous, linear-in-means peer effects are identified with a randomized treatment but linear-in-sums peer effects are not identified.

Overall, applied researchers interested in peer effects and spillovers face a variant of the usual structural trade-off. Imposing structure on the data helps identification at the risk of misspecification. With interacting agents, imposing some structure cannot be avoided (Manski, 2013). Much research is needed to understand what can be learned on peer effects and spillovers under which assumption and whether parametric models like the linear-in-means model and its extensions (see Section 2) provide good representations of the data.

## 3.3 Structural endogeneity

In the absence of a clear source of exogenous variation, can a researcher still address the problem of correlated effects and identify and estimate peer effects in networks? Researchers have developed a number of structural frameworks to do this, and we now review this growing literature. As a preliminary remark, we observe that these studies generally rely on a specific modelization of one kind of correlated effects. The proposed approach may be invalid, then, if the models are misspecified or if other kinds of correlated effects matter. Relatedly, it would be interesting to cross-validate these structural approaches with experimental data (see Griffith (2019b) for a first step in this direction).

At this stage, researchers have explored three approaches. In the first two, they combine a model of peer effects in networks with a model of network formation. A primary objective is to account for network endogeneity in the estimation of peer effects. Conversely, combining these two models may allow researchers to address endogeneity issues affecting the estimation of network formation. In a first approach, unobservable characteristics explicitly affect both individual outcomes and network formation. In a second approach, network formation itself depends on outcomes. In a third approach, omitted variables are introduced under some specific assumptions.

**Network formation and common unobservables.** Goldsmith-Pinkham and Imbens (2013) first proposed a structural approach to address the problem of correlated effects. They consider the following variant of model (5):

$$y_i = \alpha + \gamma_x x_i + \gamma_u u_i + \delta_x \bar{x}_i + \beta \bar{y}_i + \varepsilon_i \tag{6}$$

with $\mathbb{E}(\varepsilon_i|\mathbf{x},\mathbf{u},\mathbf{G}) = 0$. They introduce an unobserved characteristic, $u_i$, which may affect outcome directly but does not generate contextual peer effects. This unobserved characteristic also affects network formation, modelled through a dyadic process where undirected links are formed independently. They consider a random utility framework under the assumptions that errors' distribution is logistic and that links are formed through mutual consent. This leads to a variant of the following model.[17] Denote by $p_{ij}$ the probability that $i$ wants to form a link with $j$. Then, the link between $i$ and $j$ is formed with probability $p_{ij}p_{ji}$ with

$$ln(\frac{p_{ij}}{1-p_{ij}}) = a_0 - a_x|x_i - x_j| - a_u|u_i - u_j| \tag{7}$$

This formulation is designed to capture observed and unobserved *homophily*. Homophily is a key property of social networks, referring to the fact that similar individuals are generally more likely to be connected, (see McPherson, Smith-Lovin, and Cook, 2001). Homophily implies that the probability of link formation is higher when the distance between the characteristics of the two agents is lower. Model (7) thus displays homophily of both kinds if $a_x > 0$ and $a_u > 0$.

Authors then estimate a version of models (6) and (7) on Add Health data using Bayesian meth-

[17] Goldsmith-Pinkham and Imbens (2013) also exploit information on past connections and include a dummy for whether $i$ and $j$ were connected in the previous period and another dummy for whether they had common friends. This connects their analysis to the analysis of peer effects in networks in a panel context, see Section 3.4.

ods. They analyze grade-point average and data from one school with 534 students. In their estimation, they assume that the unobserved characteristic is binary, $u_i \in \{0,1\}$, with both values equally likely. They find that accounting for this common unobserved characteristic appears to matters for network estimation but not for peer effects. It has little impact, in particular, on the estimates of contextual and endogenous peer effects.

While implementation can be improved in many ways,[18] this approach provides a potentially powerful way to control for network endogeneity in peer effect regressions, reminiscent of Heckman's correction for sample selection. Modelling peer effects in networks and network formation simultaneously may allow researchers to recover information on common unobservables. One general limitation, of course, is that this may not help with unobservables which are not common. Potential problems of omitted variables are thus reduced but not eliminated.

In addition, we observe that identification is not very transparent and would deserve to be better understood (see Bramoullé, 2013). Identification here likely relies both on non-linearities present in the dyadic regressions and on a *functional exclusion restriction* related to homophily: the absolute value of differences in observed characteristics affect links but not outcomes. Relatedly, it is unclear whether this model *can* generate a significant bias in peer effect regressions, and precisely how this bias would be generated. A bias appears in peer effect regression if $cov(\bar{x}_i, u_i) \neq 0$, and the relation between this condition and equations (6) and (7) is not immediate. Ability to generate a bias is, of course, a key precondition to account for such a bias empirically, and this would also need to be further investigated.

Finally, Goldsmith-Pinkham and Imbens (2013) suggest that *the exogeneity of the network is testable* (see Section 6.2). This a powerful and intuitive idea, relying on homophily. Friends who are dissimilar in terms of observed characteristics must be, on average, similar in terms of unobserved characteristic. Formally, let $\eta_i$ be the residual of the peer effect regression, estimated without trying to account for the unobserved characteristic. Then under network exogeneity, $\mathbb{E}(|\eta_i - \eta_j| | |x_i - x_j| = x, g_{ij} = 1)$ does not depend on $x$ while under network endogeneity and homophily, $\mathbb{E}(|\eta_i - \eta_j| | |x_i - x_j| = x, g_{ij} = 1)$ decreases with $x$. And similar properties hold for pairs of individuals who are not connected.[19] Network exogeneity can then potentially be tested without having to estimate the combined models. Researchers have started to implement this test in applied setups (e.g. Boucher and Fortin, 2016). Indeed, we believe that testing for network exogeneity could become standard practice in applied studies of peer effects in networks. The literature is still missing, however, a

[18] For instance $u_i$ could take continuous values and could generate contextual effects, and $x_i$ and $x_j$ and $u_i$ and $u_j$ could also enter additively in equation (7).

[19] Non-friends who are similar on observed characteristics must be dissimilar on unobserved characteristics, and hence $\mathbb{E}(|\eta_i - \eta_j| | |x_i - x_j| = x, g_{ij} = 0)$ also decreases with $x$ under network endogeneity.

proper econometric analysis of this test and its statistical properties.

Researchers have extended Goldsmith-Pinkham and Imbens (2013)'s analysis in many ways. In a study developed independently, Hsieh and Lee (2016) introduce multidimensional continuous unobserved characteristics into similar combined models. They estimate their model through Bayesian methods on an Add Health sample composed of 2020 students in 73 small school-grade groups. They also analyze grade-point average and consider one, two and three unobserved characteristics. They find, interestingly, that the estimate of the endogenous peer effect is unaffected when including one unobservable, drops when including two unobservables and is then further unaffected by a third one. This suggests that introducing multiple common unobservables may be necessary to account for network endogeneity in peer effect regressions. Griffith (2019b) considers one continuous unobserved characteristic. In a first stage, he develops a dyadic model of link formation, where links are directed and *weighted*: the strength of the link is a non-negative real number. In his setup, unobserved characteristics can be identified and estimated from the network formation model alone, as in Graham (2017).[20] In a second stage, he estimates a variant of model (5) without endogenous peer effects including estimated unobserved characteristics from the first stage on the right hand side, both directly and through contextual peer effects.

Recent extensions of this approach include Hsieh and van Kippersluis (2018), Hsieh, Lee, and Boucher (2019), Auerbach (2019), Arduini, Patacchini, and Rainone (2015), Qu and Lee (2015), and Johnsson and Moon (2019) who develop econometric frameworks based on control functions, following ideas proposed by Blume et al. (2015). These studies are naturally connected to the literature on the econometrics of network formation (see Chandrasekhar, 2016; Graham, 2015; de Paula, 2017), and researchers are leveraging progress made in that literature, as in Graham (2017), to help account for the endogeneity of the network in peer effect regression.

**Network formation and outcomes.** Two recent studies propose an alternative approach to analyze peer effects in endogenous networks, by assuming that network formation is affected by outcomes (see Boucher, 2016; Badev, 2018). Boucher (2016) considers a model of conformism where agents simultaneously choose a continuous outcome $y_i \in \mathbb{R}$ and which peer to connect to. Conditional on the network, individual outcome depends on others' outcomes through a best response equation, which is a variant of model (1) without contextual peer effects. Using our notations, equation (2) in Boucher (2016) becomes:

$$y_i = \frac{\lambda d_i}{1+\lambda d_i}\bar{y}_i + \frac{1}{1+\lambda d_i}(\gamma x_i + \varepsilon_i)$$

[20] In Griffith (2019b)'s approach, the link $g_{ij}$ depends on $u_i$, $u_j$ and the product $u_i u_j$ rather than the absolute value of the difference $|u_i - u_j|$. His model thus captures complementarities, rather than homophily, in unobserved characteristics.

where $\lambda$ is a structural parameter capturing preferences for conformity. This is a particular case of an extension of model (1) where structural parameters may depend on degree, $\beta = \beta(d_i), \gamma = \gamma(d_i)$. When $\lambda > 0$, agents with a higher degree are subject to a higher endogenous peer effect $\beta$ and to a lower individual effect $\gamma$. Conditional on outcomes, links are formed through a variant of dyadic model (7) accounting for homophily in outcomes. The probability that individual $i$ wants to form a link with $j$ notably depends on observable characteristics and on $-\lambda(y_i - y_j)^2$. In this model, the parameter $\lambda$ thus controls both endogenous peer effects conditional on the network and outcome homophily in link formation. The author then derives theoretical implications of the model and develops an econometric framework based on bounds. Identification exploits the fact that changes in peers induce discontinuous changes in an individual's behavior while changes in peer behavior, holding peers fixed, induce continuous changes. He estimates his model on an Add Health sample of 30,241 individuals in the 100 smallest schools, analyzing participation in extracurricular activities. He finds that accounting for the impact of outcomes on links has little effect on the estimation of the conformism parameter, suggesting that this kind of network endogeneity matters little in this context.

Badev (2018) develops a framework where individuals simultaneously choose a binary action $y_i \in \{0, 1\}$ and peers. Conditional on the network, agent $i$'s relative utility from playing action $y_i = 1$ depends linearly on $i$'s observable characteristics, on the total number of agents playing 1 in the population, and on the difference between the number of friends playing 1 and the number of friends playing 0. The second term captures a global endogenous peer effects while the third term capture a network-based endogenous peer effect. Conditional on the actions, agent $i$'s utility to connect with $j$ depends on observable characteristics, on common neighbors and on action homophily. The second term implies that the formation of the link $ij$ depends on the presence or absence of links $ik$ and $kj$. Conditional on actions, links are then not independent and the network formation process does not reduce to dyadic regressions. This model and approach are thus related to a literature accounting for interdependencies in link formation (see Mele, 2017; Chandrasekhar and Jackson, 2018; Chandrasekhar, 2016). The third term captures homophily on outcomes: two agents who play the same action have an extra incentive to connect. As in Boucher (2016), a common parameter controls both peer effects conditional on the network and outcome homophily in network formation.

The author then develops an econometric framework based on the stationary distribution of a myopic dynamic process subject to random preference shocks, related to the frameworks of Mele (2017) and Nakajima (2007). He estimates his model on the "saturated" Add Health in-home sample of 16 schools, analyzing smoking behavior. He also finds that accounting for network formation has little impact on the estimates of the peer effect parameter. He finally performs counterfactual experiments and shows that the response of the friendship network to an increase in tobacco price amplifies the intended decrease in smoking.

**Structured omitted variables.** In a recent analysis and building on Zacchia (2019), Pereda-Fernandez and Zacchia (2019) proposes a different structural approach to address problems of correlated unobservables. Rather than considering a model of network formation, they impose some specific structure on the unobservables and on their correlations with observables in a variant of model (5). In their baseline model, they assume no contextual peer effects, homoscedasticity on $u$ and that $\mathbb{E}x_i = \lambda \sum_j h_{ij}u_j + \eta_i$ with $\mathbb{E}[\eta_i u_j] = 0$. They further assume that the econometrician knows the structure of the correlation between observed and unobserved characteristics $\mathbf{H}$ but not the extent of this correlation $\lambda$. They show that model (5) is identified under mild conditions on the interaction network $\mathbf{G}$ and the correlation network $\mathbf{H}$. For instance, if $g_{ij} = 0$ and $h_{ij} \neq 0$, the impact of $x_j$ on $y_i$ conditioning on endogenous peer effects helps identify the correlation parameter $\lambda$. They then extend their analysis and identification conditions to a more general framework.[21] While the assumption that the researcher knows the correlation structure seems strong, we believe that this analysis provides an interesting exploration of a potentially important idea. In some contexts, the structure of the network can also be used to address problems of correlated effects.

## 3.4 Panel Data

As is well-known, panel data open up identification possibilities (see Hanushek et al., 2003). The introduction of individual fixed effects, in particular, allows researchers to control for agents' time-invariant unobserved characteristics and this should help with problems of correlated effects. The literature on peer effects in networks with panel data is surprisingly scarce, however. We believe that this scarcity is due both to problems of data availability and to methodological challenges emerging when extending models (1) and (2) to a panel context. In particular, the network itself may be endogenously evolving, individual outcome may be affected by peers' time-invariant unobserved characteristics and lagged peer effects may matter. We expect network panel data to become increasingly available, however, and econometricians and applied economists should therefore devote more attention to analyze identification and estimation of panel models of peer effects in networks.

To illustrate, a natural extension of the linear-in-means model to a panel context can be written as follows:

[21] They consider a linear-in-means model with endogenous peer effects and contextual peer effects from observable characteristics, where unobservables characteristics follow a stationary Spatial Autoregressive Moving Average process function of primitive shocks and where observed characteristics are linearly correlated with these primitive shocks.

$$y_{i,t} = u_i + \delta_u \sum_j g_{ij,t} u_j + \gamma_x x_{i,t} + \delta_x \sum_j g_{ij,t} x_{j,t} + \beta \sum_j g_{ij,t} y_{j,t} + \varepsilon_{i,t} \tag{8}$$

where observed characteristics, outcomes and the network of interactions all depend on time $t$ and with $\mathbb{E}(\varepsilon_{i,t}|\boldsymbol{u}, \boldsymbol{x}_t, \boldsymbol{G}_t) = 0$. Here, $u_i$ captures time-invariant unobserved and observed characteristics of agent $i$ affecting outcomes. These time-invariant characteristics may give rise to contextual peer effects, as in model (5), captured by the parameter $\delta_u$.

In panel contexts, dynamic effects often matter and researchers may want to include various lags in the main regressions. An extension of model (8) with one-period lags can be written:

$$\begin{aligned} y_{i,t} \quad = \quad & u_i \; + \; \delta_{u,0} \sum_j g_{ij,t} u_j \; + \; \delta_{u,1} \sum_j g_{ij,t-1} u_j \; + \; \gamma_{x,0} x_{i,t} \; + \; \gamma_{x,1} x_{i,t-1} \; + \; \delta_{x,0} \sum_j g_{ij,t} x_{j,t} \\ + \quad & \delta_{x,1} \sum_j g_{ij,t-1} x_{j,t-1} \; + \; \beta_0 \sum_j g_{ij,t} y_{j,t} \; + \; \beta_1 \sum_j g_{ij,t-1} y_{j,t-1} \; + \; \theta y_{i,t-1} \; + \; \varepsilon_{i,t} \end{aligned} \tag{9}$$

In this case, individual outcome is affected by time-invariant individual characteristics, by the time-invariant characteristics of current and past peers ($\delta_{u,0}$ and $\delta_{u,1}$), by current and past individual time-varying observed characteristics ($\gamma_{x,0}$ and $\gamma_{x,1}$), by the time-varying observed characteristics of current and past peers ($\delta_{x,0}$ and $\delta_{x,1}$), by the outcomes of current and past peers ($\beta_0$ and $\beta_1$) and by past individual outcome ($\theta$).

At this stage in the literature, no study has analyzed or applied full-fledged versions of model (8) or model (9). All studies consider particular cases, missing important ingredients. In a group context, Arcidiacono et al. (2012) consider a model where individual outcome is only affected by individual and peers' time-invariant characteristics.[22] They propose a non-linear least square estimator and show that if peer groups change over time, and under some further assumptions, the estimator does not run into an incidental parameter problem and yields consistent estimates of $\delta_u$. It would be interesting to see whether their analysis extends to network interactions.

A few papers analyze peer effects in networks with panel data, see Patnam (2013), De Giorgi, Frederiksen, and Pistaferri (2019) and Comola and Prina (2019).[23] Patnam (2013) analyzes the effect of corporate networks on firms' investment and executive pay using panel data for publicly traded companies in India. She assumes that a firm's reference group is composed of all other firms

[22] Their model is a case of model (8) with no impact of individual and peers' observed time-varying characteristics and with no endogenous peer effect, $\gamma_x = \delta_x = \beta = 0$.

[23] In other studies, researchers assume that they do not know the network and exploit panel data to identify peer effects and the network structure itself, see Section 4.

with whom it shares a board director. She considers shocks on the board interlock network caused by directors' retirement or death. She discusses how endogenous peer effects can be identified if these shocks on the network are exogenous, a variant of the random peers arguments presented in Section 3.1. These three studies introduce individual fixed effects but do not consider contextual peer effects associated with these time-invariant characteristics. We view this as a potentially important limitation of their frameworks.

# 4 Imperfect knowledge of the network

In the previous Sections, we assumed that the researcher knows the true network of interactions between agents. This is a strong assumption, which may not hold in reality and for different reasons. The researcher may have information on a sample of the population and on links between them, but not on individuals outside the sample and on their connections. The network in the data may be measured with errors. More generally, even if a specific network of relationships is well measured, how to be sure that this is the relevant network of interactions with respect to the outcome considered? For instance, many studies analyze peer effects on grade-point average among high school students based on self-reported friendships. Are self-reported friends necessarily the relevant peers with respect to academic achievement?

We believe that imperfect knowledge of the network of interactions is a first-order empirical issue which deserves more attention. For instance, difficulties raised by incomplete sampling appear in every analysis of in-home data from Add Health on the non-saturated sample and are generally neglected. Misspecifying the network can, of course, invalidate the identification and estimation of peer effects based on the network structure. As pointed out by Blume et al. (2015), incorrectly assuming an absence of connections invalidates the exclusion restriction for instrumental variables built from the network structure. *When the network is mismeasured, peers of peers who are not peers in the observed network can, in fact, be peers in the real network.* Their characteristics then have a direct impact on individual outcome and cannot be used to instrument for friends' outcomes in peer effect regressions. This issue is particularly relevant given the documented tendency of social networks towards clustering and transitivity: friends' friends are generally quite likely to have some form of direct relationship.

How to ensure, then, an absence of direct connections between indirectly connected agents? Applied economists have developed an ingenious potential solution to the problem, by combining information from different networks. In their study of peer effects on mothers' working hours, Nicoletti, Salvanes, and Tominey (2018) consider both family peers - sisters and female cousins - and geographic neighbors. In their main regression, they instrument the working hours of family peers by the past working hours of the family peers' neighbors. In their study of peer effects on consumption, De Giorgi, Frederiksen, and Pistaferri (2019) combine information on spouses and on coworkers. They instrument coworkers' consumption by shocks on the firms of coworkers of the spouses of these coworkers. Juxtaposing different dimensions of the social space helps ensure that indirectly connected agents are not directly connected.[24] This is a conservative approach: researchers focus on parts of the observed social structure where intransitivity and the related exclusion restrictions are likely to hold.

More generally, a promising idea emerging from the recent literature is that peer effects can be identified even with very imperfect knowledge of the actual network of interactions, see in particular Theorems 6 and 7 in Blume et al. (2015). In a panel context, peer effects can even be identified with no knowledge of the network under the assumption that the network does not change over time, see Manresa (2016), de Paula, Rasul, and Souza (2018), Rose (2016). In that case, the network of interactions itself can potentially be identified and be recovered from the data. Network time-invariance is of course a strong assumption, unlikely to hold in many contexts. However, panel data are not necessarily needed to identify and estimate peer effects with imperfect knowledge of the network. Boucher and Houndetoungan (2019) propose methods to estimate models (1) and (2) with cross-sectional data when the network is imperfectly observed but the researcher can consistently estimate the network's probability distribution.

Sampling and measurement error are two important and underresearched topics. Chandrasekhar and Lewis (2016) provide an econometric analysis of sampled networks (see Conti et al. (2013) for an early contribution). They consider two sampling schemes. The researcher observes a random sample of nodes and either their connections with other sampled nodes or their connections with all other nodes. They notably show that in regressions based on model (1), instruments built from friends of friends who are not friends in the *observed* network are generally invalid, see their Proposition 3.4. To see why, suppose that $i$ is connected to $j$ and $l$ who are both connected to $k$. Suppose also that $i$, $j$ and $k$ are sampled but $l$ is not. Even though $i$ and $k$ are not directly connected, $x_k$ also affects $y_i$ through its impact on the unobserved $y_l$ and the corresponding exclusion restriction is not satisfied. They also propose a simple analytical correction for the second scheme when the researcher also observes the outcomes and characteristics of all agents. Griffith (2019a) explores the implications of *degree censoring*, i.e. imposing upper bounds on the number of peers when eliciting network data, on the estimation of model (1). He shows that degree censoring is widespread in practice, as with Add Health data (Jackson, 2013). He finds that this can significantly bias the estimates

[24]Relatedly, Brollo, Kaufmann, and Ferrara (2018) find that individuals' compliance responds to penalties incurred by siblings' classmates, in a study on learning spillovers about the enforcement of a social program.

and discusses strategies to address this issue. Overall, much more research is needed to understand the statistical and econometric implications of network sampling, network measurement error and, more generally, imperfect network knowledge on peer effect regressions.

# 5 Concluding remarks

An important theme which emerges from our survey is the adaptability of the identification strategy based on the network structure: peers of peers who are not peers affect individual outcome only through their effect on peers' outcomes. This strategy can be applied directly with a randomized treatment even when the network is endogenous, if the network is not affected by the treatment; it can be applied to identify endogenous peer effects with random peers even when contextual peer effects are not identified; it can be adapted to account for correlated effects in some contexts. This strategy may not be valid, however, when the network is mismeasured and agents with no apparent connection are, in fact, peers. In empirical applications, researchers have notably combined information from different networks to help ensure an absence of connections between peers of peers.

The literature on peer effects in networks is growing fast, with no lack of important open questions. There is, for instance, still very little research on panel data (Comola and Prina, 2019), on measurement error (Chandrasekhar and Lewis, 2016), or on combining structural and experimental approaches (Griffith, 2019b). Perhaps the most challenging open question, however, concerns the mechanisms behind peer effects. How to disentangle the roles of conformity, complementarities, social learning, risk sharing and other motives behind peer effects? Recent progress on this question has been made thank to well-designed experiments (Beugnot et al., 2019; Breza and Chandrasekhar, 2019) and to structural estimation of theoretical models (Banerjee et al., 2013). More generally, we believe that the development, analysis and empirical estimation of new theoretical models of network interactions will prove key to identify the reasons behind peer effects. While future research will undoubtedly open up new perspectives, we conjecture that the key insight that the network structure contains useful information for causal identification will prove long lived.

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