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Cross-Examination

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ABSTRACT

Two opposed parties seek to influence an uninformed decision maker. They invest in acquiring information and select what to disclose. The decision maker then adjudicates. We compare this benchmark with a procedure allowing adversarial cross-examination. A cross-examiner tests the opponent in order to persuade the decision maker that the opponent is deceitful. How does the opportunity or threat of cross-examination affect the parties' behavior? How does it affect the quality of decision-making? We show that decision-making deteriorates because parties are less likely to acquire information and because cross-examination too often makes the truth appear as falsehood. Next, we consider a form of controlled cross-examination by permitting the cross-examined to be re-examined by his own advocate, i.e., counter-persuasion. More information then reaches the decision maker. Decision-making may or may not improve compared to the benchmark depending on how examination is able to trade off type 1 and 2 errors.

Keywords: Bayesian persuasion, disclosure game, adversarial, redirect examination, procedural rules.

JEL codes: C72, D71, D82, D83, K41

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1 Introduction

Adversarial cross-examination is a key feature of the common law trial. More or less constrained forms are often also allowed in other institutions, e.g., industrial tribunals, disciplinary panels, regulatory hearings, commercial arbitration boards. How does the opportunity or threat of cross-examination affect the acquisition and submission of evidence by interested parties? How does it affect the quality of decision-making? We consider a persuasion game where two parties with opposed interests search for evidence in order to influence an uninformed decision maker. The potential evidence consists of many pieces of hard information from which each party will select what to disclose. The decision maker then adjudicates. We compare this benchmark setting with a procedure allowing cross-examination, by which we mean raising issues about the other party's report. Cross-examination elicits information as to whether the opponent was misleading through withholding of evidence, enabling the decision maker to update her belief about the significance of a report. Next, we introduce the opportunity of re-examination whereby a cross-examined party is questioned by his own advocate. The purpose is to mitigate a possibly unfavorable cross-examination outcome.

We find that cross-examination does not by itself improve the quality of decision-making. Everything else equal, given a rational (i.e., Bayesian) decision maker, it has no effect on the probability of correct adjudication. However, because it is conducted in a partisan manner, it benefits the cross-examining party on average. Cross-examining the opponent or submitting countervailing evidence are substitutes in potentially countering the opponent. A party benefitting from the opportunity of cross-examination therefore has less incentives to gather hard information about the fact at issue. As a result, the quality of inferences from the opponent's possibly deceitful report will deteriorate. This is because the quality of these inferences also depend on the likelihood that the cross-examining party itself acquired information that it did not disclose. In addition, there is a chilling effect on the gathering of evidence by the party threatened by cross-examination. The option value not to disclose unfavorable information is reduced and there is the risk of erroneously appearing to be deceitful following cross-examination. That party will therefore be less likely to come forward with relevant evidence. Altogether, conditional on the acquisition of evidence, cross-examination would not affect the quality of decision-making, but from the decision maker's standpoint there is now too little gathering of evidence. When the opportunity of re-examination is introduced, to control the excesses of cross-examination, more information reaches the decision maker. Nevertheless, compared to the benchmark, decision-making may or may not improve depending on how cross and re-examination are able to trade off type 1 and 2 errors, i.e., making the truth appear as falsehood versus not detecting actual deceitfulness. The next section provides an example illustrating some of the dynamics at play.

Strictly speaking, cross-examination is the interrogation of a witness called by the adverse party after the witness has been subject to direct examination by that party. This is one of the main differences between the common law adversarial procedure and its counterpart in the civilist tradition. In the latter, as concerns civil disputes, it is the parties' responsibility to provide evidence but there is no or little adversarial cross-examination. Most of the questioning comes from the bench, that is, nonpartisan examination. In the common law tradition, cross-examination is widely believed to be indispensable. In Wigmore's much quoted phrase, it "is beyond any doubt the greatest legal engine ever invented for the discovery of truth."(Wigmore, 1940, § 1367). Cross-examination has also been criticized: "Wigmore's celebrated panegyric...is nothing more than an article of faith." (Langbein, 1985, p. 834). Most of the criticism focuses on cross-examination's potential for 'false positives', i.e., the cross-examined erroneously appears to be deceitful. Judge Frankel remarked that cross-examination is "like other potent weapons, equally lethal for heroes and villains" (Frankel, 1975, p. 1039); and that a skillful cross-examiner "will employ ancient and modern tricks to make a truthful witness look like a liar." (Frankel, 1980, p. 16). Indeed, less well known than Wigmore's famous quotation is the caveat that shortly follows it: "A lawyer can do anything with cross-examination...He may...do more than he ought to do; he...may make the truth appear like falsehood."(Wigmore, 1940, § 1367).

The caveat recognizes the need for cross-examination to be controlled.

Across legal systems and decision-making institutions, there is a wide variety of procedural rules governing examination, including nonpartisan examination and restrictions on the scope of cross or re-examination. In some hybrid judicial systems, parties question witnesses first, then judges ask additional questions; in other systems, the presiding judge will ask questions first, then parties may be allowed to question the witnesses (see Weigend, 2010). In the US over recent years, a debate developed about whether adversarial cross-examination should be allowed in Title IX hearings about sexual assault cases on college campuses. Opponents argued that adversarial cross-examination would deter reporting of sexual assault and impose undue toll on complainants, while proponents emphasized the right to due process for respondents. One proposal was to allow the accused party to submit cross-examining questions to a neutral examiner who would select the questions most likely to shed light on the case.[1]

Some legal scholars remarked that there is no theory – and little empirical evidence – about how cross-examination works.[2] We will interpret cross-examination in the general sense of actions that seek to lessen the weight of another party's report not by providing directly relevant countervailing evidence but by questioning the report's significance or interpretation. Viewed as part of a truth-finding enterprise, it is said that cross-examination delves into the reliability of testimonies, e.g., the witness exhibits confusion or unwillingly misstates facts; or it seeks to clarify narrative ambiguities or the meaning of words or concepts in a submission. A cross-examiner may also question the credibility of a testimony, e.g., the expert contradicts himself and perhaps lies. As most practitioners would acknowledge, however, cross-examination and similarly for re-examination is not about finding the truth

[1] In 2011 the Obama administration substantially revised Title IX grievance procedures in order to encourage reporting. In 2020, the Trump administration issued its own Title IX guidance, in the name of enhancing due process protection for accused students. The controversial new requirement concerned adversarial cross-examination. See Behre (2020) and Dowling (2021), and the many references therein.

[2] See Sanchirico (2009). To quote Wigmore himself: "What is the theory of [cross-examination's] efficiency?...Upon this we commonly reflect but little." (ibid, § 1368). See also Lempert: "[T]he likely effectiveness of cross-examination in getting at the truth is seldom examined – numerous court opinions and commentaries rely on Wigmore's conclusion...rather than on empirical evidence." (Lempert, 1998, p. 345).

as such, but about winning. In our analysis, both the cross and re-examiner are partisans seeking to maximize their probability of prevailing.

As a start, we follow the economic literature on disclosure games in assuming that evidence is hard information that can be concealed but cannot be falsified or forged (Grossman 1981, Milgrom 1981, Milgrom and Roberts 1986). In this framework, the intrinsic meaning of pieces of information is assumed to be common knowledge and outright lies are not possible. A report may nevertheless be deceitful because of the withholding of information. When an interested party's information status is common knowledge, full revelation of private information is induced by the decision maker's skeptical posture of assuming the worst. Full unraveling fails, however, when there is a possibility that the party possesses no hard information (Dye 1985, Shavell 1989). Competition between parties with opposed interests is then generally beneficial to the decision maker (see for instance Bhattacharya and Mukherjee, 2013).

We integrate the possibility of cross-examination into the disclosure game framework. Our basic set-up is similar to Shin's (1998) analysis of the adversarial procedure but with the following features. First, we consider situations where the parties' information is endogenous, as in Kim (2014) or Kartik et al. (2017). Secondly, the information acquired by a party may consist of several pieces (Dewatripont and Tirole, 1999; Demougin and Fluet, 2008; or Bull and Watson, 2019). The two features together allow for equilibria where both parties invest in acquiring information and where a party may sometimes rebut the other party. Moreover, when a party submits evidence, the decision maker may remain uncertain whether the party disclosed the whole truth. The latter feature allows for actions, referred to as cross-examination or re-examination as the case may be, that influence the interpretation of reports by eliciting information, possibly noisy, about the eventual withholding of evidence.

Cross-examination is modeled as a 'publicly observable experiment' in the spirit of the Bayesian persuasion literature (Kamenica and Gentzkow, 2011; Kamenica, 2019; Bergemann and Morris, 2019). The cross-examiner subjects the cross-examined to a testing process. The test is designed to maximize the probability that the cross-examined will fail the test, subject to Bayesian plausibility and informational constraints restricting the set of feasible tests. Re-examination is modeled similarly except that the re-examiner has the opposite objective. When the procedure allows both cross and re-examination, there is persuasion and counterpersuasion as in models of Bayesian persuasion with multiple senders (Gentzkow and Kamenica, 2017a, 2017b; Li and Norman, 2021).

Our analysis also relates to the literature on voluntary versus mandatory disclosure. It is well known that reducing the scope of manipulating information may be detrimental to the quality of decision-making when information is costly and the uninformed party is a sophisticated Bayesian (Matthews and Postlewaite, 1985; Farrell, 1986; Shavell, 1994; Schweizer, 2017). There is then a trade-off between the agent's incentives to acquire information and the quality of communication conditional on the information acquired. Our results differ because, owing to the adversarial context, the weight that should be given to a possibly deceitful report depends on how likely both parties are informed. In addition, rather than mandatory disclosure as such, we deal with the involuntary transmission of information. What information is transmitted is strategically distorted through partisan persuasion. It may also be tilted by the set of feasible tests. For instance, it may be easier to suggest that a report is deceitful than to persuade that it is wholly truthful.

The paper develops as follows. The next section presents a simple motivating example. Section 3 describes the basic set-up. Section 4 analyzes the procedure without the opportunity of cross-examination, which serves as benchmark. Section 5 derives the main results by first allowing cross-examination and then both cross and re-examination. Section 6 concludes. Proofs are in the Appendix.

2 An Example

An individual files a medical malpractice suit. He underwent a treatment that sometimes results in complications causing harm. In figure 1a, the adverse outcome is unlikely, with probability $\eta = 0.025$ if care is efficient but always arises if care is inefficient, by which we mean errors in diagnosis, mistakes in performance, and the like. The legal rule is that the plaintiff is entitled to damages if it can be shown that harm was most likely due to inefficient care, i.e., a probability greater than one half. The overall frequency of inefficient care is $p = 0.05$.[3] Applying Bayes' rule, the posterior probability of inefficient care conditional on an adverse outcome is

$$\mu = \frac{p}{p + (1-p)\eta} = 0.678 \tag{1}$$

On this basis, absent the possibility of other sources of information, the plaintiff would prevail.

However, suppose that an adverse outcome can also result from some particular precondition. Prior to the treatment, our individual did not know that this precondition posed a risk; this information was obtained in the course of filing suit. The individual may not even know whether he has this precondition. Figure 1b describes the plaintiff's information status. With probability $\theta = 0.5$, some years back, he faced events that necessitated check-ups. For the case at hand, the relevant information from these check-ups is that they establish whether the plaintiff had or did not have a precondition. The probability of a precondition is $\eta = 0.025$.

There are therefore three possible types of plaintiffs: uninformed ones with no past record, informed ones with a record showing no precondition, and informed ones with a record showing a precondition. A record showing a precondition is unfavorable to the plaintiff.[4] An informed plaintiff's strategy is therefore to submit the good record and to withhold the bad one; uninformed ones submit no record. To complete the setting, we add the possibility that the defendant may uncover the plaintiff's record from years back. When a record exists, the probability that the defendant uncovers the record is $e = 0.7$. An informed defendant's strategy is to submit the record if it shows a precondition, as this will counter the plaintiff's claim. Otherwise the defendant stays put. Taking all this into account, when no record

[3] This is for the sake of our example. Danzon (1991) reports studies showing that a similar percentage of hospitalized patients suffered complications caused by health care management.

[4] Conditional on an adverse outcome, the probability of inefficient care is then the prior $p = 0.05$, much less than the more-likely-than-not threshold.

is submitted by either the plaintiff or the defendant, the court's posterior belief that care was inefficient is

$$\mu = \frac{(1-\theta)p + \theta p\eta(1-e)}{(1-\theta)[p+(1-p)\eta] + \theta\eta(1-e)} = 0.62 \tag{2}$$

The denominator is the joint probability of an adverse event and that no record is submitted. The numerator is the joint probability of no record and inefficient care.

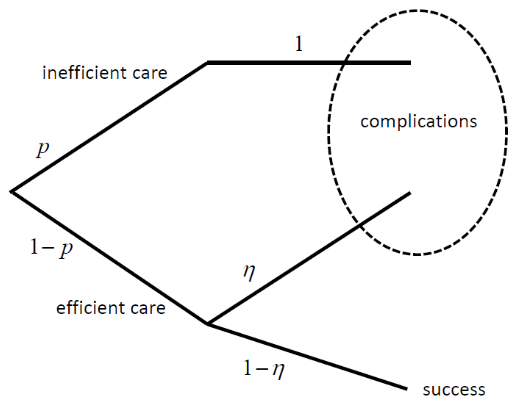

Fig. 1a. Medical malpractice case ($p = .05$, $\eta = .025$)

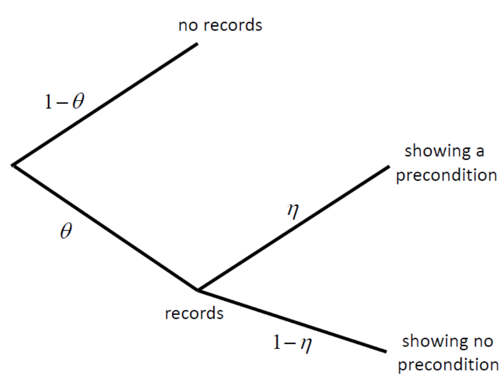

Fig 1b. Plaintiff's information status ($\theta = .5$, $\eta = .025$)

To summarize, the plaintiff prevails unless the defendant submits a record showing a precondition. This describes our benchmark setting. Table 1 shows the probability that the plaintiff prevails per adverse event and the probability of judicial error per adverse event.

We now augment the defendant's toolbox by allowing him to cross-examine the plaintiff. The purpose of cross-examination is to elicit information about the plaintiff's information status. This is useful when the plaintiff submitted no report and the defendant also did not. A cross-examination strategy is a line of questioning. Two strategies, A and B, are assumed to be available to the defendant, each one yielding two possible outcomes, *good* or *bad* from the plaintiff's point of view. The probabilities of these outcomes depend on the plaintiff's information status and the chosen strategy. For both strategies, the outcome is *bad* with probability one when the plaintiff is informed, i.e., when he concealed evidence. The outcome is *bad* with probability α when the plaintiff is uninformed (a 'false positive'), where α depends on the cross-examination strategy. For strategy A, $\alpha = 0.257$; for strategy B, $\alpha = 0.923$. Why we picked these values will become clear shortly. Notice that the *good* outcome perfectly reveals that the plaintiff is uninformed, so that the plaintiff then prevails. On the other hand, when no medical record is submitted by either party and the outcome of cross-examination is *bad*, the court's posterior belief that care was inefficient is

$$\mu = \frac{(1-\theta)p\alpha + \theta p\eta(1-e)}{(1-\theta)[p + (1-p)\eta]\alpha + \theta\eta(1-e)} \tag{3}$$

The court observes the line of questioning and therefore knows what cross-examination strategy the defendant is following. It uses the appropriate α to update its belief.

If the defendant were to use strategy B, the court's posterior would be $\mu = 0.616$ which would be of no use to the defendant. Strategy B is too noisy to sufficiently influence the court's belief. However, with strategy A, the court's posterior following the *bad* cross-examination outcome is $\mu = 0.5$. The defendant then prevails because inefficient care is not more likely than not.[5] Therefore, the defendant chooses strategy A. As a result, the plaintiff now prevails only when he can submit a record showing no precondition

[5] Strategy A is analogous to the prosecutor's best Bayesian persuasion strategy in Kamenica and Gentzkov's (2011) motivating example.

or when the outcome of cross-examination is *good*. The implications are shown in the second line of table 1, referred to as the interim case. Notice that the opportunity of cross-examination benefits the defendant because the plaintiff prevails less often, but that it has no effect on the probability of judicial error. Compared to the situation without cross-examination, the error merely shifts from one kind (erroneously finding for the plaintiff) to the other kind (erroneously finding for the defendant).

However, the interim case is not an equilibrium because the defendant can still do better. Searching for the plaintiff's past records (which may not even exist) involved investigation costs. The defendant now realizes that uncovering a precondition record is superfluous. It suffices to cross-examine the plaintiff. In our example, this always leads to the *bad* cross-examination outcome when the plaintiff withheld evidence, which is precisely when an informed plaintiff did not submit the record because it showed a precondition. The defendant therefore switches to no investigation and relies solely on cross-examination to counter the plaintiff. Although the court may not observe the defendant's evidence gathering, it will understand the defendant's incentives. As a result, equation (3) with $e = 0.7$ no longer describes the court's posterior belief. Setting $e = 0$ in this expression, the appropriate belief is

$$\mu = \frac{(1-\theta)p\alpha + \theta p\eta}{(1-\theta)[p + (1-p)\eta]\alpha + \theta\eta} \tag{4}$$

With the cross-examination strategy A, the court's belief is now $\mu = 0.301$ so that the defendant again prevails following the *bad* cross-examination outcome. The belief is more unfavorable to the plaintiff because the court has become more skeptical vis-à-vis the plaintiff when no record is submitted. Owing to the court's greater skepticism, it turns out that the cross-examination strategy B now performs better from the defendant's point of view. With this strategy, the court's posterior as computed in (4) is now $\mu = 0.5$. Strategy B is preferred by the defendant because it does as well when the plaintiff is informed and much better when he is uninformed. Thus, when cross-examination is allowed, the defendant will not investigate at equilibrium and he will use the cross-examination strategy B. The implications are shown in the third line of table 1. The plaintiff prevails less often than when cross-examination is not allowed and the probability of judicial error is larger.

Table 1
An example

	False positive rate under cross-examination	Probability[1] that plaintiff prevails	Probability[1] of judicial error
Equilibrium without cross-examination	- -	0.881	0.209
Interim situation: cross-examination strategy A	0.257	0.702	0.209
Equilibrium with cross-examination: strategy B	0.923	0.369	0.334

[1]The probability is per adverse event

3 Model

An arbitrator adjudicates an issue between two parties referred to as the plaintiff P and the defendant D. The issue is the value of $\omega \in \{\omega_0, \omega_1\}$ or true fact, where ω_0 favors the defendant and ω_1 favors the plaintiff. The true fact is unknown and all share the same prior probability p that it is ω_1. The arbitrator must make a binary decision $d \in \{0, 1\}$ where 0 means that she finds for the defendant and 1 that she finds for the plaintiff. She wants her decision to match the true fact. Her payoff is $u_A(d, \omega) = 1$ if $d = i$ when $\omega = \omega_i$, where $i \in \{0, 1\}$, and $u_A(d, \omega) = 0$ otherwise. Each party wants the arbitrator to rule in his favor. The plaintiff's payoff from the arbitrator's decision is d, the defendant's payoff is $-d$.

Investigation. There is uncertainty about the potential pool of information, e.g., related facts, documents, or witnesses. With probability $\theta \in (0, 1)$, the evidence contains two pieces of hard information, denoted x and y with realizations in $\{x_0, x_1\}$ and $\{y_0, y_1\}$ respectively. With probability $1 - \theta$, the evidence consists of the single piece x. For instance, some document is known to exist, but there is uncertainty about the existence of yet another relevant document. Similarly, it is known that there was a witness of some event pertaining to the issue, but there is uncertainty about the presence of a second witness or whether there is a video recording of the event. m denotes the state where the evidence consists of the two pieces x and y; n is the state where the evidence consists of the single piece x. That there are two possible states of the evidence is common knowledge.

Searching for the evidence is costly and not always successful. Party $j \in \{P, D\}$ uncovers the evidence with probability e_j at a cost $C(e_j)$, an increasing and strictly convex function with $C(0) = C'(0) = 0$ and $C'(1) \geq 1$. The inequality ensures that, given the stakes, a party will never want to obtain the evidence for sure. When the state of the evidence is m, a successful party uncovers (x, y); otherwise, a successful party uncovers only x. Given the investment in gathering evidence, the parties' net payoffs are

$$u_P = d - C(e_P), \quad u_D = -d - C(e_D).$$

How the evidence relates to the true fact is as follows. $P(x, y, \omega)$ denotes the joint probability when the state of the evidence is m. We will also use P to denote marginal or conditional distributions derived from $P(x, y, \omega)$; for instance, the prior is $P(\omega_1) = p$. The realizations of x and y are independent conditionally on the true fact,

$$P(x, y \mid \omega) = P(x \mid \omega)P(y \mid \omega). \tag{5}$$

From Bayes' rule, the posteriors are

$$P(\omega_i \mid x, y) = \frac{P(x, y, w_i)}{P(x, y, \omega_1) + P(x, y, \omega_0)} \qquad i = 0, 1.$$

The following assumption ensures the existence of equilibria where the plaintiff bears the burden of proof.

Assumption 1: $p \leq \frac{1}{2}$, $P(y_1 \mid w_1) \geq P(y_0 \mid w_0) > \frac{1}{2}$, and x and y are sufficiently informative for the posteriors to satisfy the inequalities in table 2.

Table 2
Posterior probabilities

	$P(\omega_1 \mid x)$	$P(\omega_1 \mid x, y_0)$	$P(\omega_1 \mid x, y_1)$
x_0	$< \frac{1}{2}$	$< \frac{1}{2}$	$> \frac{1}{2}$
x_1	$> \frac{1}{2}$	$< \frac{1}{2}$	$> \frac{1}{2}$

When the state of the evidence is n, the joint density of x and ω is $P(x, \omega)$; that is, x has the same meaning irrespective of the state of the evidence.

Communication and adjudication. Investigation is followed by a communication phase in which the parties may report to the arbitrator. Reports are denoted by r_j, $j \in \{P, D\}$. If party j was unsuccessful in obtaining the evidence, his submission is by force the empty report $r_j = \emptyset$. If the party was successful and the state of the evidence is n, his report belongs to the set $\{\emptyset, (x, \emptyset)\}$ where $\emptyset$ means that he submits nothing and $(x, \emptyset)$ that he reports only x. If the party was successful and the state of the evidence is m, his report belongs to the set $\{\emptyset, (x, \emptyset), (\emptyset, y), (x, y)\}$. Thus, when a party reports nothing, the arbitrator does not know whether the party was truly unsuccessful or whether he chose not to submit evidence. When a party reports $(x, \emptyset)$, the arbitrator does not know whether the party could also have submitted y. When a party reports $(\emptyset, y)$, however, the state of the evidence is revealed and the arbitrator knows that the party could have submitted x as well.

The foregoing is sufficient to describe a procedure without the opportunity of cross-examination, which we take as benchmark. The time line is then as follows. First, Nature chooses the true fact, whether the evidence consists of one or two pieces, and the realizations of the pieces of evidence, all of which remains unobservable at this stage. Next, the parties simultaneously choose their investigation efforts, e_P and e_D respectively, and Nature chooses whether they access the evidence or not, all of which is private information.[6] At the third stage, the parties simultaneously submit their reports r_P and r_D. At the last stage, the arbitrator observes the reports, updates her beliefs, and adjudicates. The solution concept is perfect Bayesian equilibrium.

We write $\mu(r_P, r_D)$ for the arbitrator's updated belief that the true fact is ω_1. Similarly, her adjudication strategy is $d(r_P, r_D) \in \{0, 1\}$. The sequentially rational decision is $d = 1$ if her belief $\mu > \frac{1}{2}$ and $d = 0$ if $\mu < \frac{1}{2}$. When $\mu = \frac{1}{2}$, she is indifferent between finding for one party or the other. As tie-breaker, we impose that she then finds for the defendant. The plaintiff prevails only if the arbitrator believes that ω_1 is more likely than not. We confine attention to equilibria with $\mu(\emptyset, \emptyset) \leq \frac{1}{2}$, meaning that the plaintiff bears the burden of proof in the sense that he loses when no evidence is submitted.[7]

Cross-Examination. When cross-examination is allowed, an additional stage is inserted between disclosure and adjudication. A party submitting evidence can now be questioned by the adverse party. Cross-examination yields information that differs from the direct evidence discussed so far because it relates to the possibility that a party manipulated his report. The cross-examiner seeks to elicit whether the party knows more than he reported, for instance another relevant document was uncovered but was not submitted.[8] In the adjudication phase, the arbitrator's belief will now depend on the parties' reports and on the outcome of cross-examination. We will also consider the possibility of re-examination, also known as redirect examination. The counsel of the cross-examined party then attempts to mitigate the consequences of cross-examination by himself putting questions to that party. This generates additional information, possibly allowing a more favorable interpretation of the party's answers to the cross-examiner. We defer the details to section 4.

[6] The parties would have less incentives to acquire evidence if their investigation effort were observable by the arbitrator. See Henry (2009) and Wong and Yang (2018).

[7] This is without loss and has no impact on the insights.

[8] We assume that there is no penalty for withholding evidence. Legal advice is routinely about the selection of information to disclose to a tribunal (Kaplow and Shavell, 1989; Che and Severinov, 2017). Actually destroying evidence is another matter; see Sanchirico (2004) and Bull (2009).

Remarks on the assumptions. The asymmetry between the informativeness of x and y may seem arbitrary. As will become clear, when both x and y were equally informative, so that y_0 can only offset or nullify x_1, the opportunity of cross-examination has no bearing on decision-making. What matters for our results is the possibility that evidence possibly concealed by one party may *rebut* (rather than simply nullify) the party's report, where by rebuttal we mean that the posterior switches from above to below one half.

The same property can be obtained in a purely symmetric setting. For instance, let the set of possible pieces of evidence be $X = \{x, x', x''\}$, where the signals are independent conditionally on the true fact and have the same precision q. Specifically,

$$P(z_i \mid \omega_i) = q > \frac{1-p}{p} \quad z \in \{x, x', x''\},\ i \in \{0, 1\}$$

where the inequality ensures that $P(\omega_i \mid z_i) > \frac{1}{2}$. The possible states of the evidence is then the collection of non empty subsets of X, so that there are more than two states. When the pool of evidence is the full X and, say, the plaintiff only disclosed the realization x_1, the disclosure of x'_0 and x''_0 by the defendant would rebut x_1. In our setting, y_0 plays the same role as the pair (x'_0, x''_0) while avoiding the complexity of more than two states of the evidence.

We assumed that, when the parties uncover evidence, they access exactly the same pieces depending on the state of the evidence. Alternatively, it could be that the parties are independently more or less lucky in what they find. For instance, one party could uncover x while the opponent uncovers both x and y, these events being uncorrelated. This would not affect our results but the exposition is slightly more involved because of the greater number of relevant events.[9]

Finally, we assumed that the adjudicator seeks to maximize the probability of correct adjudication., i.e., she puts equal weights on the two possible types of error (erroneously finding for the plaintiff of erroneously finding for

[9] This was the approach in an earlier version of this paper.

the defendant). Alternatively, she could seek to minimize a weighted sum of errors. This would not affect our analysis except for the obvious changes.

4 No Cross-Examination

Disclosure strategies. Equilibria may differ in inessential ways with respect to disclosure decisions. We consider a profile of strategies consistent with both the cases where cross-examination is or is not allowed. The plaintiff is proactive because he is the party bearing the burden of proof. He always submits a priori favorable evidence and suppresses a priori unfavorable evidence, where 'a priori' refers to the raw posteriors in table 1. By contrast, the defendant has a minimum disclosure strategy. He only submits reports containing y_0, if he can.

The plaintiff's behavior is referred to as a 'sanitization strategy' in Shin (1994); see also Shin (1998) and Kartik et al. (2017). In these settings, each party can only access a single piece of evidence which is either disclosed whole or not at all. In our case, the evidence may come in one or two pieces. Disclosing only one piece reveals that the party was successful in uncovering the evidence. This may create suspicion that the other piece has been withheld, i.e, a party's disclosure conveys information through the report's *face value* and as a signal of the party's private information (Bull and Watson 2019). This subjects the party to the risk of adverse inferences or of cross-examination when the procedure allows it. The defendant's strategy of only submitting reports containing the overpowering y_0 avoids this risk.

The plaintiff's disclosure strategy is

$$\begin{cases} (\emptyset,\emptyset),(x_0,\emptyset),(x_0,y_0) & \longmapsto & \emptyset \\ (x_1,\emptyset),(x_1,y_0) & \longmapsto & (x_1,\emptyset) \\ (x_0,y_1) & \longmapsto & (x_0,y_1) \\ (x_1,y_1) & \longmapsto & (x_1,y_1) \end{cases} \tag{6}$$

The defendant's strategy is

$$\begin{cases} (\emptyset,\emptyset),(x_0,\emptyset),(x_1,\emptyset),(x_0,y_1),(x_1,y_1) & \longmapsto & \emptyset \\ (x_0,y_0) & \longmapsto & (x_0,y_0) \\ (x_1,y_0) & \longmapsto & (x_1,y_0) \end{cases} \tag{7}$$

When the plaintiff observes (x_0, y_1), he discloses the whole evidence rather than $(\emptyset, y_1)$ even though x_0 is by itself unfavorable. Should he do the latter, he would reveal that he has also observed x. At worst, the arbitrator then infers that the evidence is (x_0, y_1). But then the arbitrator would anyway rule for the plaintiff; see table 2. Thus, it does not matter whether the plaintiff discloses (x_0, y_1) or $(\emptyset, y_1)$. A similar argument applies for the defendant's disclosure of (x_1, y_0) rather than $(\emptyset, y_0)$.[10]

Many pairs of reports never arise on the equilibrium path. The arbitrator's beliefs are then obtained as the limit of completely mixed strategies where the parties play out-of-equilibrium moves with some small probability. Beliefs off the equilibrium path yield the same decisions as would the raw posteriors in table 2.

Investigation strategies. From the foregoing, the arbitrator rules in the plaintiff's favor if the evidence is (x_0, y_1) or (x_1, y_1) and possibly also if it reduces to x_1. So far, the arbitrator's strategy $d(r_P, r_D)$ is therefore completely defined except following the pair of messages $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$.

At the investigation stage, the parties' expected payoffs are

$$\overline{u}_P = V(e_D)e_P - C(e_P), \quad \overline{u}_D = -V(e_D)e_P - C(e_D), \tag{8}$$

where

$$V(e_D) \equiv (1-\theta)P(x_1)d + \theta[P(y_1) + P(x_1, y_0)(1 - e_D)d] \tag{9}$$

is the probability that the plaintiff prevails conditional on having access to the evidence and d is short-hand for $d((x_1, \emptyset), \emptyset)$.

The expression in (9) follows from the disclosure strategies. When $d = 1$, the plaintiff prevails, conditional on having uncovered the evidence, if (i) the state of the evidence is n and $x = x_1$; or if (ii) the state of the evidence is m and either $y = y_1$ or $(x, y) = (x_1, y_0)$ and the defendant did not uncover the evidence, which occurs with probability $1 - e_D$. When $d = 0$, the plaintiff

[10] There will be equilibria where the defendant does not investigate and therefore will not be in a position to report along the equilibrium path. The strategy (7) then describes what he would do should he investigate and obtain evidence.

prevails only if the state of the evidence is m and $y = y_1$.

In (8), e_D is the plaintiff's conjecture of the defendant's investment; e_P is the defendant's conjecture of the plaintiff's investment. At equilibrium, the conjectures are correct and investigation efforts are mutual best-responses given the arbitrator's strategy. There are then two possibilities.

(i) Passive Defendant equilibrium. When $d((x_1, \emptyset), \emptyset) = 0$, the conditional probability that the plaintiff succeeds is $V(e_D) = \theta P(y_1)$. The defendant gains nothing from gathering evidence while the plaintiff's investigation effort solves

$$C'(e_P) = \theta P(y_1). \tag{10}$$

The investigation efforts in the *Passive Defendant* equilibrium, henceforth *pd*-equilibrium, are denoted by e_P^{pd} and e_D^{pd}.

(ii) Active Defendant equilibrium. When $d((x_1, \emptyset), \emptyset) = 1$, the conditional probability that the plaintiff succeeds is

$$V(e_D) = (1 - \theta)P(x_1) + \theta[P(y_1) + P(x_1, y_0)(1 - e_D)]. \tag{11}$$

Searching for evidence is now profitable for both parties. The equilibrium investigation efforts solve the system of first-order conditions:

$$C'(e_P) = (1 - \theta)P(x_1) + \theta[P(y_1) + P(x_1, y_0)(1 - e_D)], \tag{12}$$

$$C'(e_D) = e_P \theta P(x_1, y_0). \tag{13}$$

We denote by e_P^{ad} and e_D^{ad} the investigation efforts in the *Active Defendant* equilibrium, henceforth *ad*-equilibrium.

Lemma 1 *When these equilibria exist, (i) there is a unique Passive Defendant equilibrium with* $e_P^{pd} > e_D^{pd} = 0$*; (ii) there is a unique Active Defendant equilibrium with* $e_P^{ad} > e_D^{ad} > 0$*.*

For the *ad*-equilibrium, the defendant's best response investigation effort is increasing in e_P while the plaintiff's is decreasing in e_D, so there is a unique solution.[11] In both types of equilibria the plaintiff is more likely to be the

[11] In a contest model, Katz (1988) obtains that the expenditure of the ex ante favored

better informed party.

Arbitrator's beliefs and equilibria. The critical belief is the one associated with $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$. This pair of reports cannot arise when the state of the evidence is m and the defendant also uncovered the evidence. If $y = y_1$, the plaintiff would have reported it; if $y = y_0$, the defendant would have reported it. However, these reports may arise if the state of the evidence is n or if it is m and the defendant was unsuccessful. The arbitrator weighs these possibilities. Her skepticism vis-à-vis the plaintiff depends on her conjecture of the defendant's investigation effort. To make this explicit, we write the arbitrator's belief as $\mu((x_1, \emptyset), \emptyset; e_D)$ where e_D is the arbitrator's conjecture.

Lemma 2 $\mu((x_1, \emptyset), \emptyset; e_D) > \frac{1}{2}$ *is equivalent to*

$$k_P(1-\theta) > k_D\theta(1-e_D) \tag{14}$$

where

$$k_P \equiv P(x_1, \omega_1) - P(x_1, \omega_0) > 0,\ k_D \equiv P(x_1, y_0, \omega_0) - P(x_1, y_0, \omega_1) > 0.$$

Condition (14) always holds if $\theta < \theta_a \equiv k_P/(k_P + k_D)$ *and otherwise if*

$$e_D > \varphi(\theta) \equiv 1 - \left(\frac{k_P}{k_D}\right)\left(\frac{1-\theta}{\theta}\right). \tag{15}$$

In (14), k_P is the value of finding for the plaintiff on the basis of the sole evidence x_1, i.e., it is the probability that ruling in the plaintiff's favor is the correct decision minus the probability of error. Similarly, k_D is the value of finding for the defendant on the basis of the evidence (x_1, y_0) that the defendant would have reported had he been able to. That k_P and k_D are positive follows from assumption 1. In particular, k_D is positive because y is more informative than x.

When only the plaintiff submits evidence, the arbitrator leans towards

party is 'deterring' while that of the underdog is 'provocative'; see also Daughety and Reinganum (2000). We obtain a similar characterization with the underdog as the party bearing the burden of proof.

the plaintiff if the prior probability θ that the potential evidence consists of two pieces is not too large or if the defendant's investigation effort is sufficiently large. A large e_D makes it more likely that there is only a single piece of evidence. The arbitrator's belief that the state of the evidence is m, equivalently that evidence was retained, is equal to

$$\nu(e_D) = \frac{\theta P(x_1, y_0)(1 - e_D)}{(1-\theta)P(x_1) + \theta P(x_1, y_0)(1 - e_D)} \tag{16}$$

which is decreasing in e_D. The arbitrator's belief about the true fact at issue incorporates (16) but also takes into account the relative informativeness of the pieces of evidence.

Proposition 1 *An equilibrium with $\mu(\emptyset, \emptyset) < \frac{1}{2}$ always exists. When $\theta < \theta_a$, it is the ad-equilibrium. When $\theta \geq \theta_a$, it is either the pd-equilibrium which then always exists or the ad-equilibrium which also exists provided θ is not too large.*

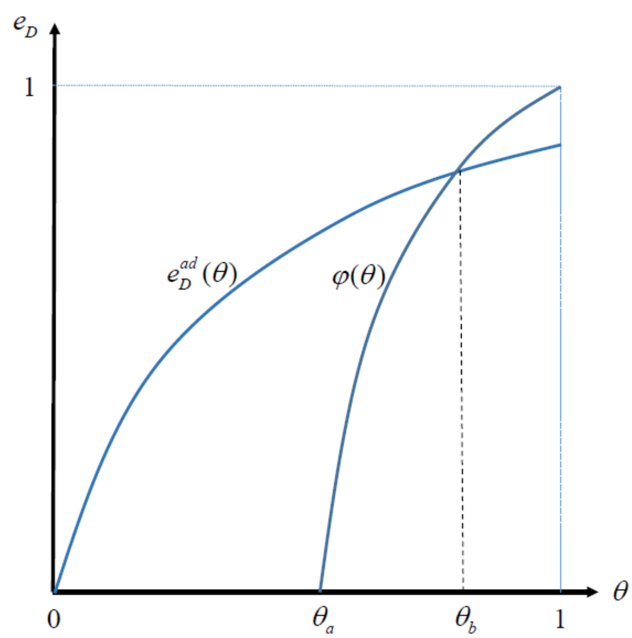

Fig. 2. Active and passive defendant equilibria

In figure 2, $e_D^{ad}(\theta)$ is the defendant's investigation effort in the solution to the system (12)-(13); hence, $e_D^{ad}(0) = 0$ and $e_D^{ad}(\theta) < 1$ for all θ. From lemma 2, $\varphi(\theta)$ is an increasing concave function with $\varphi(\theta_a) = 0$ and $\varphi(1) = 1$. The curves $e_D^{ad}(\theta)$ and $\varphi(\theta)$ therefore intersect, which occurs at $\theta = \theta_b$ in the figure. As drawn, when $\theta \geq \theta_b$, the unique outcome is the *pd*-equilibrium because the conditions of lemma 2 do not hold. When $\theta < \theta_a$, these conditions always hold and the unique outcome is the *ad*-equilibrium. When $\theta \in [\theta_a, \theta_b)$, both types of equilibria exist. In the *ad*-equilibrium, when the plaintiff submits x_1 and the defendant is silent, the arbitrator finds for the plaintiff because $e_D^{ad}(\theta) > \varphi(\theta)$. In the *pd*-equilibrium, the arbitrator finds for the defendant because $e_D^{pd} = 0 < \varphi(\theta)$. We cannot rule out the possibility that the curves cross more than once. Henceforth, we take θ_b to be the smallest value at which the curves intersect and restrict attention to situations where $\theta < \theta_b$.

Quality of decision-making. The arbitrator's expected utility, equivalently the probability of correct adjudication is

$$\begin{aligned} \overline{u}_A &= p \Pr(d = 1 \mid \omega_1) + (1-p) \Pr(d = 0 \mid \omega_0) \\ &= 1 - p + [\Pr(d = 1, \omega_1) - \Pr(d = 1, \omega_0)] \,. \end{aligned} \tag{17}$$

where $\Pr(d \mid \omega)$ is the probability of decision d at equilibrium, conditional on the true fact being ω, and $\Pr(d, \omega)$ is the joint probability. Different equilibria, and different procedures as when cross-examination is introduced, will differ in the quality of decision-making only through the expression in brackets.

In the *pd*-equilibrium, equation (17) becomes

$$\overline{u}_A^{pd} = 1 - p + e_P^{pd} \Lambda^{pd} \tag{18}$$

where

$$\Lambda^{pd} \equiv \theta \left[P(y_1, \omega_1) - P(y_1, \omega_0) \right]. \tag{19}$$

When no evidence is communicated, the probability of correct adjudication equals $1 - p \geq \frac{1}{2}$, merely on the basis of the burden of proof assignment. The next term in equation (18) is the value added by the investigation and communication phases. We refer to Λ^{pd} as the value of communication in the *pd*-equilibrium. It is the improvement in decision-making that results from the potential communication of y_1.

In the *ad*-equilibrium,

$$\overline{u}_A^{ad} = 1 - p + e_P^{ad}\Lambda^{ad} \tag{20}$$

where

$$\Lambda^{ad} = \Lambda^{pd} + \left[k_P(1-\theta) - k_D\theta(1-e_D)\right]. \tag{21}$$

From lemma 2, the expression in brackets is positive when the *ad*-equilibrium exists. The value of communication Λ^{ad} is then the improvement in decision-making that results from the potential communication of y_1 or of $(x_1, \emptyset)$.

Proposition 2 *When both the pd and the ad-equilibria exist, the latter yields a smaller probability of error. The plaintiff more often submits evidence,* $e_P^{ad} > e_P^{pd}$*, and communication is more informative,* $\Lambda^{ad} > \Lambda^{pd}$*.*

The *ad*-equilibrium leads to better decision-making for two reasons. First, some evidence is more likely to be communicated because the plaintiff investigates more. Secondly, the value of communication is greater. The arbitrator's inferences are now more 'informed' because the defendant also investigates. This improves decision-making because the arbitrator is less skeptical, allowing the plaintiff to succeed when the only evidence submitted is x_1.

5 Procedures with Examination

Cross-examination can serve a purpose only when the plaintiff prevails if he reports $(x_1, \emptyset)$ and the defendant does not submit counterevidence. This arises when the outcome would have been the *ad*-equilibrium absent the opportunity of cross-examination.

To determine the effect of examination, whether only cross or both cross and re-examination, it suffices to represent the outcome as a binary signal correlated with the plaintiff's information status. This suffices because the arbitrator's decision is itself binary. As a first step, we take the properties of the signal as given. Next, we make the signal endogenous and study the strategic choices of the cross-examiner or of both the cross and re-examiner.

5.1 Active Examination Equilibria

Following the reports $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$, the defendant (or his counsel) may decide to cross-examine the plaintiff. This triggers a process, possibly also including re-examination, that yields the signal $\chi \in \{b, g\}$ where b and g denote the good and bad outcomes from the plaintiff's perspective. The probabilities are

$$\beta \equiv \Pr(\chi = b \mid m) \text{ and } \alpha \equiv \Pr(\chi = b \mid n),$$

where α is the type 1 error (the false positive rate) and β is the power of the test (the true positive rate), equivalently $1 - \beta$ is the type 2 error (false negative rate).

Examination allows the arbitrator to update her belief about the state of the evidence, equivalently whether the plaintiff reported the whole truth. We assume that either $\alpha > 0$ or $\beta < 1$, meaning that examination is a noisy test. Applying Bayes' rule,

$$\hat{\theta}_b \equiv \Pr(m \mid \chi = b) = \frac{\theta\beta}{\theta\beta + (1-\theta)\alpha} \tag{22}$$

$$\hat{\theta}_g \equiv \Pr(m \mid \chi = g) = \frac{\theta(1-\beta)}{\theta(1-\beta) + (1-\theta)(1-\alpha)} \tag{23}$$

with $\hat{\theta}_b > \theta > \hat{\theta}_g$. These are raw posteriors that differ from the arbitrator's equilibrium beliefs about the state of the evidence because the latter also takes into account the defendant's investigation effort. The equilibrium beliefs are given by (16) with $\hat{\theta}_b$ or $\hat{\theta}_g$ in lieu of θ, depending on the realization of χ.

Strategies and payoffs. Let σ denote the defendant's cross-examination strategy. When he chooses not to cross-examine, we write $\sigma = \emptyset$; when he does, we write $\sigma = \chi$ because the arbitrator will then observe the realization of χ at the end of the examination process. In the adjudication phase, the arbitrator's belief about the fact at issue is $\mu((x_1, \emptyset), \emptyset, \sigma)$.

Although allowed, examination need not arise. First, it may be that the plaintiff loses in the absence of cross-examination, so the defendant gains nothing by cross-examining. The outcome is then the *pd*-equilibrium, as when cross-examination is not allowed. Secondly, it may be that, when the plaintiff prevails in the absence of examination, he also does irrespective of the outcome of examination because it is insufficiently informative. Cross-examination then does not matter, so we adopt the convention that no examination is triggered. The outcome is then the *ad*-equilibrium.[12] Finally, there is the case where examination is triggered because it potentially benefits the defendant who hopes that the plaintiff will fail the test. This yields the *Active Examination* equilibrium, henceforth the *ae*-equilibrium.

In this equilibrium, the plaintiff fails the test if the examination outcome is $\chi = b$. This occurs with probability α when the state of the evidence is n, equivalently when the plaintiff did not retain evidence, and with probability β when the state of the evidence is m. Conditional on uncovering the evidence, the probability that the plaintiff prevails is now

$$V(e_D, \alpha, \beta) \equiv (1-\theta)P(x_1)(1-\alpha)+\theta[P(y_1)+P(x_1, y_0)(1-\beta)(1-e_D)]. \quad (24)$$

The parties' investigation efforts solve the system of first-order conditions:

$$C'(e_P) = (1-\theta)P(x_1)(1-\alpha) + \theta[P(y_1) + P(x_1, y_0)(1-\beta)(1-e_D)], \quad (25)$$

$$C'(e_D) = e_P\theta(1-\beta)P(x_1, y_0). \quad (26)$$

We denote by e_P^{ae} and e_D^{ae} the solution to system (25)-(26).

Lemma 3 *The system (25)-(26) has a unique solution satisfying $e_P^{ae} > e_D^{ae} \geq 0$. Moreover, $e_D^{ae} < e_D^{ad}$.*

The defendant always investigates less compared to the *ad*-equilibrium. As before, the plaintiff is more likely to be the better informed party.

[12] The *pd* and *ad*-equilibria are defined as before except that the definition also specifies the examination strategy and the arbitrator's belief as a function of the outcome of that strategy.

Beliefs. Replicating the approach in section 3, we characterize the conditions for the arbitrator's beliefs to be consistent with an *ae*-equilibrium. The beliefs must support the decision $d((x_1, \emptyset), \emptyset, \emptyset) = 1$, i.e., the plaintiff prevails when he reports $(x_1, \emptyset)$ and is not cross-examined, together with the decision $d((x_1, \emptyset), \emptyset, b) = 0$, i.e., the plaintiff loses if he fails the examination. As before, the beliefs depend on the arbitrator's conjecture of the defendant's investigation effort.

Lemma 4 $\mu((x_1, \emptyset), \emptyset, b; e_D) \leq \frac{1}{2} < \mu((x_1, \emptyset), \emptyset, \emptyset; e_D)$ *if and only if*

$$k_P(1-\theta) > k_D\theta(1-e_D) \tag{27}$$

and

$$k_P(1-\hat{\theta}_b) \leq k_D\hat{\theta}_b(1-e_D), \tag{28}$$

equivalently

$$e_D \leq \hat{\varphi}(\theta) \equiv 1 - \left(\frac{\alpha}{\beta}\right)\left(\frac{k_P}{k_D}\right)\left(\frac{1-\theta}{\theta}\right). \tag{29}$$

The condition (27) is the same as in lemma 2 and ensures that the plaintiff prevails when there is no cross-examination. The condition (28) is an inference constraint in terms of the posteriors conditional on the unfavorable examination outcome, ensuring that the plaintiff then loses. Given any positive α, $\hat{\varphi}(\theta)$ is negative for sufficiently small values of θ. The inference constraint (28) then cannot be satisfied. The constraint cannot be satisfied when $k_D = 0$, except in the limiting case where the type 1 error is nil.[13] Assumption 1 ensures that $k_D > 0$, which is equivalent to

$$P(w_1 \mid x_1, y_0) = \frac{P(x_1, y_0, w_1)}{P(x_1, y_0, w_0) + P(x_1, y_0, w_1)} < \frac{1}{2},$$

meaning that y_0 rebuts x_1 as defined in section 3.

[13] In this limiting case, $\hat{\theta}_b = 1$ so that (28) always holds.

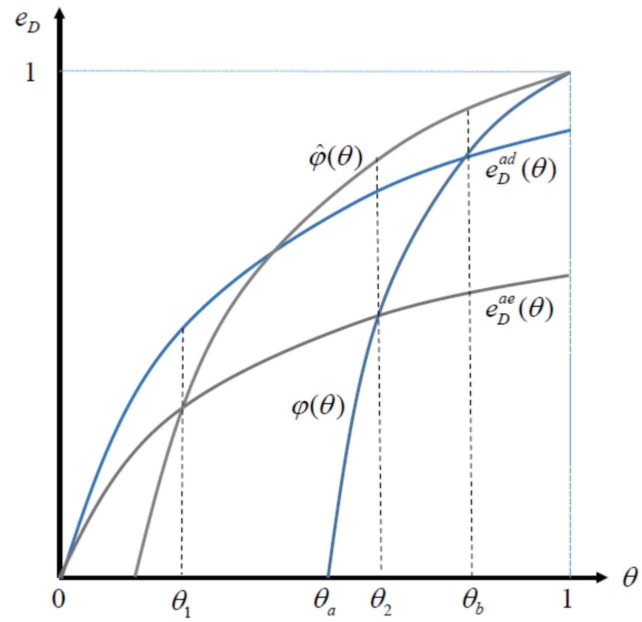

Fig. 3. Equilibria with examination opportunity when $\alpha > 0$

In Figure 3, the curves $\varphi(\theta)$ and $e_D^{ad}(\theta)$ are the same as before. The curve $\hat{\varphi}(\theta)$ is defined by (29), assuming $\alpha > 0$, and $e_D^{ae}(\theta)$ is part of the solution to (25)-(26). From lemma 3, $e_D^{ae}(\theta)$ is everywhere below the $e_D^{ad}(\theta)$ curve. When the procedure does not allow examination, an *ad*-equilibrium exists for all $\theta < \theta_b$. In the interval $[\theta_2, \theta_b)$, when examination is allowed, an *ad*-equilibrium does not exist because condition (28) is satisfied with $e_D = e_D^{ad}(\theta)$. At the investigation phase, the parties would be mistaken in their expectation that the plaintiff prevails for sure when only x_1 is submitted. However, neither can we have an *ae*-equilibrium because condition (27) is not satisfied with $e_D = e_D^{ae}(\theta)$. The possibility of countering the plaintiff through examination reduces the defendant's incentives to investigate. As a result, the arbitrator is more skeptical vis-à-vis the plaintiff when only x_1 is submitted. The only remaining possibility is therefore the *pd*-equilibrium.

5.2 Cross-Examination Only

Cross-examination unfolds before the arbitrator who draws the appropriate inferences from the outcome of the test to which the plaintiff is subjected. For instance, following a string of so-called leading questions which the cross-examined can only answer with yes or no, that party may be led to contradict himself or the outcome may at least suggest contradiction. How much information can be extracted from the plaintiff through such a test depends on the cross-examiner's skill or ingenuity and on the particularities of the situation. We formalize this as constraints on the space of feasible experiments.

Examination set. Let Z be a random variable with density f_n if the state of the evidence is n and density f_m if the state of the evidence is m.

Assumption 2: The support of f_n is the interval $[z_n, 1]$ with $z_n \geq 0$, that of f_m is $[0, z_m]$ with $z_n < z_m \leq 1$; the densities are continuous over their supports and $f_m(z)/f_n(z)$ is strictly decreasing over (z_n, z_m).

The assumption is the monotone likelihood ratio (MLR) property with the possibility that the supports do not perfectly overlap.[14]

The reunion of the supports is the unit interval. Experiments are represented as finite partitions of this interval.[15] Thus, an experiment is a finite collection $\{\pi(s)\}_{s\in S}$ of disjoint sets (with positive measure) such that $\cup_{s\in S}\{\pi(s)\} = [0,1]$ where S indexes the sets in the partition. Equivalently, an experiment defines a signal χ with realizations in S and conditional probabilities

$$\Pr(\chi = s \mid j) = \int_{\pi(s)} f_j(z)\, dz, \quad j \in \{m, n\},\, s \in S. \tag{30}$$

The examination set is the space of such experiments or signals.

As noted earlier, of particular interest are the binary signals with $S = \{b, g\}$. These are defined by (Borel measurable) functions $\psi : [0,1] \to \{0,1\}$ such that

$$\alpha \equiv \int_0^1 \psi(z) f_n(z)\, dz, \quad \beta \equiv \int_0^1 \psi(z) f_m(z)\, dz. \tag{31}$$

[14] The MLR property is without loss of generality given that there are only two states, m and n.

[15] The partition representation is due to Green and Stokey (1978) and has been used in multisender Bayesian persuasion models, e.g., Gentzkov and Kamenica (2017b), Li and Norman (2021). We differ slightly from these models because we restrict the space of feasible signals.

A cross-examination strategy with binary signal amounts to choosing a function $\psi(z)$.

From basic statistics and given the MLR property, for any $z_1 \geq z_n$, the strategy $\psi(z) = 1$ if $z \leq z_1$, $\psi(z) = 0$ otherwise maximizes β for the achieved level of type 1 error.[16] Because the MLR property is strict, the upper bound, denoted $\hat{\beta}(\alpha)$, is an increasing and strictly concave function so long as $\hat{\beta}(\alpha) < 1$. How much information can be extracted from the plaintiff depends on this upper bound. For instance, when $z_m < 1$, the cross-examiner can achieve $\beta = 1$ with some $\alpha < 1$. The interpretation is that a plaintiff who concealed evidence will fail this test for sure, although there is still a risk of type 1 error. Even a truthful plaintiff may contradict himself. Conversely, when $z_n > 0$, a positive β can be achieved without any type 1 error. For instance, there may be a line of questioning that only catches the villain. A larger upper bound $\hat{\beta}(\alpha)$ means that more informative experiments (in the sense of Blackwell) are feasible.

Equilibria. We first inquire whether the *ad* outcome can be sustained without triggering cross-examination. Using (22), the inference constraint (28) can be rewritten as

$$\alpha k_P(1-\theta) \leq \beta k_D \theta (1 - e_D)$$

where α and β are given by (31) for some function ψ. When cross-examination is allowed, the following condition is sufficient to rule out the *ad* outcome.

Condition I. $\alpha k_P(1-\theta) \leq \beta k_D \theta (1 - e_D^{ad})$ for some feasible (α, β), $\beta > 0$.

The *ad*-equilibrium then does not exist because, due to the opportunity of cross-examination, the defendant prevails with positive probability when the plaintiff reports $(x_1, \emptyset)$ and the defendant provides no counterevidence. The outcome is then either the *ae* or the *pd*-equilibrium. Given (27) and assuming $\alpha > 0$, condition I implies

$$\frac{\beta}{\alpha} \geq \frac{k_P(1-\theta)}{k_D \theta (1 - e_D^{ad})} > 1,$$

[16] This is Neyman-Pearson's lemma. See Lehmann and Romano (2005).

meaning that effective cross-examination must be sufficiently more lethal for villains than heroes, contrary to the quote by Frankel (1975) in the introduction.

Consider now the ae-equilibrium. The information set defined by $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$ can be reached in two circumstance: (i) the defendant did not access the evidence; or (ii) the defendant accessed the evidence and the state of the evidence is n, i.e., the plaintiff did not in fact withhold evidence. It is plausible that the cross-examiner, acting for the defendant, shares the same information as the defendant. Thus, the cross-examiner may be of two types, either uninformed or informed, where the latter means that he knows that the potential evidence reduces to the single piece disclosed by the plaintiff.

This raises the possibility that the cross-examination strategy chosen by the defendant reveals his information status. In a related framework, Perez-Richet (2014) considers a setting with a binary type space, an informed sender, and a binary decision space for the receiver. He shows that one can confine attention to pooling equilibria. In our setting, these are equilibria where both the informed and uninformed cross-examiner choose the same cross-examination test.[17] Moreover, Perez-Richet shows that plausible refinements select the 'high type' optimal communication strategy. In our setting the high type is the uninformed cross-examiner. An informed defendant would not want his type to be revealed as this would reveal that the plaintiff did not suppress evidence and thereby defeat the purpose of cross-examination. We therefore consider cross-examination strategies that an uninformed defendant would choose. As it turns out, an informed one will be also be happy with such strategies.

An uninformed cross-examiner does not know whether the state of the evidence is m or n. He therefore shares the same belief as the arbitrator at this stage of the game, as defined in (16) with $e_D = e_D^{ae}$. The cross-examination test is designed to maximize the probability that the plaintiff

[17] The pooling result does not necessarily obtain when the sender's payoff is a continuous function of the receiver's ex post beliefs; see Hedlund (2017).

fails the test. This solves

$$\max_{\psi} \nu(e_D^{ae})\alpha + (1 - \nu(e_D^{ae}))\beta,$$

where α and β satisfy (31) subject to the inference constraint

$$\alpha k_P(1-\theta) \leq \beta k_D \theta (1 - e_D^{ae}). \tag{32}$$

Because $e_D^{ae} < e_D^{ad}$, condition I ensures that the constraint set defined by (32) is not empty. The solution to the above problem is $\beta = \hat{\beta}(\alpha)$ where α solves

$$\frac{\hat{\beta}(\alpha)}{\alpha} = \frac{k_P(1-\theta)}{k_D\theta(1-e_D^{ae})} \tag{33}$$

The solution is unique because $\hat{\beta}(\alpha)/\alpha$ is strictly decreasing in α.

Notice that the equilibrium cross-examination strategy is an efficient test, i.e., the power of the test is maximized for the achieved level of type 1 error. In effect, within the set of most powerful tests, the cross-examiner chooses the one with maximum type 1 error subject to the inference constraint (32).

Proposition 3 *Let $\theta < \theta_b$ so that the ad-equilibrium exists when cross-examination is not allowed. Then, when it is allowed and condition I holds, the outcome is either (i) an ae-equilibrium with $e_P^{ae} > e_D^{ae} \geq 0$ solving (25)-(26) and with $\beta = \hat{\beta}(\alpha)$ and α solving (33); or (ii) it is the pd-equilibrium with $e_P^{pd} > e_D^{pd} = 0$ and no cross-examination. If $\theta < \theta_a$, the outcome is as in (i); if $\theta \geq \theta_a$, the outcome is as in (ii) when the examination set is sufficiently rich.*

In Figure 4a, condition I is satisfied by any pair (α, β) between line D and $\hat{\beta}(\alpha)$, so that an *ad*-equilibrium is ruled out. The cross-examination strategy defined by α^* satisfies condition (33). The slope of E is smaller than that of D because $e_D^{ae} < e_D^{ad}$ as shown in lemma 3. Figure 4b is a similar example, except that $\hat{\beta}(\alpha^*) = 1$. Such a situation may arise when $z_m < 1$, so that the binary signal can be partially revealing.

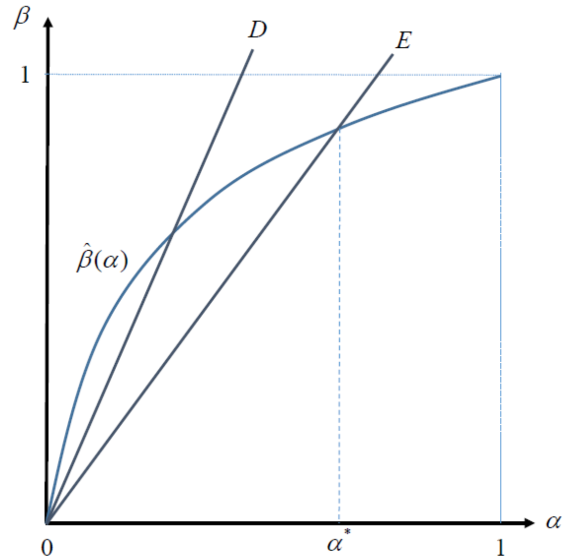

Fig. 4a. Cross-examination with $\widehat{\beta}(\alpha^*) < 1$

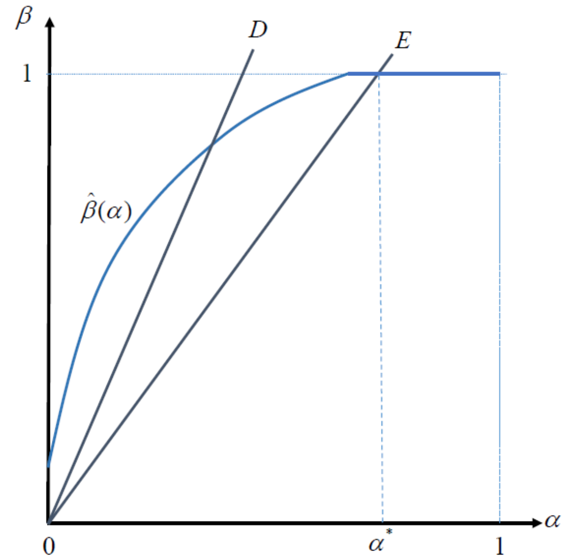

Fig. 4b. Cross-examination with $\widehat{\beta}(\alpha^*) = 1$

A proviso is that the pair of investigation efforts (e_P^{ae}, e_D^{ae}) together with α^* is consistent with an ae-equilibrium only if condition (27) of lemma 4 is satisfied with $e_D = e_D^{ae}$. This is always the case when $\theta < \theta_a$, but need not be so when $\theta \geq \theta_a$. For instance, when $\hat{\beta}(\alpha^*) = 1$ as in figure 4b, the solution to (25)-(26) implies $e_D^{ae} = 0$. Hence, (27) is not satisfied for $\theta \geq \theta_a$. It follows that the outcome is then the *pd*-equilibrium. By continuity, a similar argument can be made when $\hat{\beta}(\alpha^*) < 1$ and is sufficiently large.

5.3 Cross and Re-Examination

The procedure now allows the plaintiff's counsel to re-examine the plaintiff after cross-examination. The goal is counterpersuasion, to repair the damages from an unfavorable cross-examination. Re-examination is constrained to only address issues raised on cross-examination. This is typical of most procedures. We assume that the re-examiner is equally skillful. He has access to the same set of signals and can choose a signal arbitrarily correlated with that of the cross-examiner.[18] This means that the re-examiner decides what additional (feasible) information will be revealed at each realization of the cross-examiner's signal. For instance, a schooled cross-examiner will have learned not to ask the "one question too many" and not to allow the cross-examined to explain his answers. The re-examiner may ask the "one question too many", enabling the party to clarify his testimony.

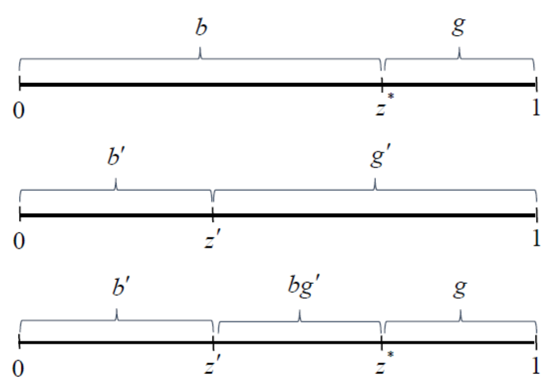

Fig. 5. Persuasion and counterpersuasion

To illustrate, consider the outcome represented in figure 4a. Because the cross-examiner's test is efficient, the bad outcome b reveals that the

[18]The possibility to perfectly correlate signals is the approach used in Gentzkov and Kamenica (2017a) and Li and Norman (2021).

realization of Z belongs to some interval $[0, z^*]$ with upper bound solving $\int_0^{z^*} f_n(z)\,dz = \alpha^*$. As counterpersuasion, the re-examiner can conduct an experiment generating a binary signal whose bad outcome b' reveals $[0, z']$, where $z' < z^*$. The joint signals from cross and re-examination then amount to the signal with realizations b', bg', and g as shown in figure 5. Alternatively, the re-examiner could have directly chosen the latter signal. Either way, he can always refine whatever the cross-examiner does.

Now, when α^* satisfies (33), there will exist some z' such that the outcome bg' favors the plaintiff. This follows from the strict concavity of $\hat{\beta}(\alpha)$, so that

$$\frac{f_m(z^*)}{f_n(z^*)} = \hat{\beta}'(\alpha^*) < \frac{\hat{\beta}(\alpha^*)}{\alpha^*} = \frac{k_P(1-\theta)}{k_D\theta(1-e_D^{ae})}$$

Therefore, for z' not too small,

$$k_P(1-\theta)\int_{z'}^{z^*} f_n(z)\,dz > k_D\theta(1-e_D^{ae})\int_{z'}^{z^*} f_m(z)\,dz.$$

The preceding inequality implies that, when the outcome is bg', the arbitrator finds for the plaintiff because it is sufficiently likely that no evidence was suppressed. The cross-examiner's strategy depicted in figure 4a will then be partially defeated, implying that the situation cannot be an equilibrium. In choosing his investigation effort, the defendant would be overestimating his chances of prevailing when he triggers examination.

Equilibria. We now characterize the strategies in the continuation game following the reports $r_P = (x_1, \emptyset)$ and $r_D = \emptyset$. The relevant stages are (i) the defendant chooses a cross-examination test; (ii) the plaintiff's counsel observes the cross-examination test and its outcome and chooses a re-examination test; (iii) the arbitrator observes the cross and re-examination tests, their realizations, and adjudicates.

Step (ii) raises a concern similar to one discussed earlier. Because the plaintiff knows the state of the evidence, his counsel plausibly also does. By itself, the chosen re-examination strategy could therefore signal the plaintiff's type, i.e., whether it is m or n. Borrowing from Perez-Richet (2014) once more, we restrict attention to pooling equilibria where both types of re-examiner choose the same re-examination test. The high type is the type-n re-examiner because a type-m would not want his type to be revealed, as this would defeat the purpose of re-examination. We therefore consider re-examination strategies that type n would choose.

As described in (ii), the re-examiner observes the outcome of cross-examination before choosing his own strategy. As shown in Li and Norman (2021), when signals can be arbitrarily correlated, it is without loss of generality to analyze sequential persuasion as if signal realizations were observed only after all players have chosen their communication strategies. It is also without loss to focus on *one-step* equilibria, where non-trivial information is generated only by the first sender's communication strategy. What other senders would do only acts as constraints on the first-sender's communication strategy, much like incentive compatibility constraints. In the present context, a one-step equilibrium is a situation where the re-examiner has no incentives to refine the test conducted by the cross-examiner.

Thus, the cross-examiner chooses a test that maximizes the probability of disfavoring the plaintiff, subject to the constraint that the test will not be defeated in the sense that the re-examiner has no incentive to add non-trivial information. Formally, the cross-examiner now solves

$$\max_{\psi} \nu(e_D^{ae})\alpha + (1 - \nu(e_D^{ae}))\beta$$

subject to the inference constraint (32) and to the additional constraint

$$\psi(z)[k_P(1-\theta)f_n(z) - k_D\theta(1-e_D^{ae})f_m(z)] \leq 0, \quad \text{all } z \in [0,1]. \tag{34}$$

If (34) is not satisfied over some set with positive measure, then over this set we must have $\psi(z) = 1$ together with

$$k_P(1-\theta)f_n(z) > k_D\theta(1-e_D^{ae})f_m(z).$$

If this were so, a type-n re-examiner would want to generate a signal that potentially reveals realizations in this set, because this favors the plaintiff. Observe that a type-m counsel would not necessarily want to do so because it may be that $f_m(z) = 0$ over this set, meaning that it would not strictly benefit the type-m plaintiff; see the discussion of figure 5c below. As argued above, however, at equilibrium both types of re-examiners pool on the same strategy.

Proposition 4 *When both cross and re-examination are allowed, the outcome is as in Proposition 3 except that, in the ae-equilibrium, the defendant's cross-examination strategy induces $\beta = \hat{\beta}(\alpha)$ where α maximizes*

$$\hat{\beta}(\alpha) k_D \theta (1 - e_D^{ae}) - \alpha k_P (1 - \theta).$$

Compared to the procedure without the opportunity of re-examination, the defendant investigates more and the examination test has a smaller type 1 error; it also has less power, except possibly when $\beta = 1$ in both procedures.

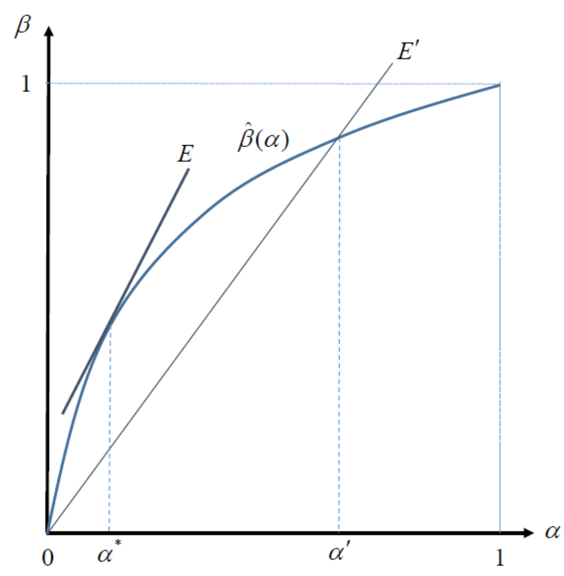

Fig. 6a. Cross and re-examination: interior solution

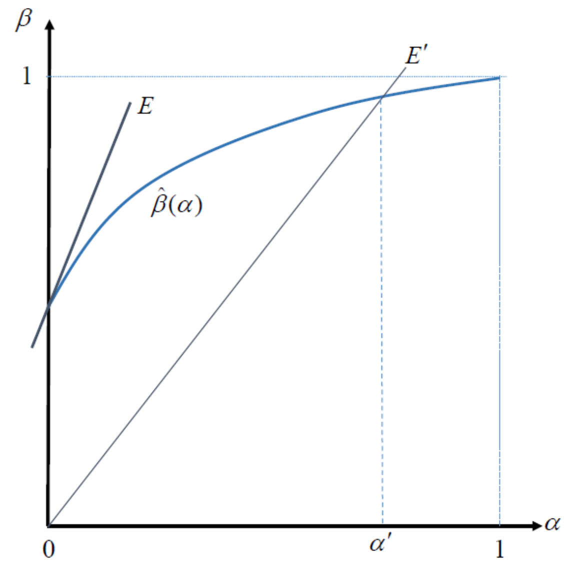

Fig. 6b. Cross and re-examination: corner solution with $\alpha^* = 0$

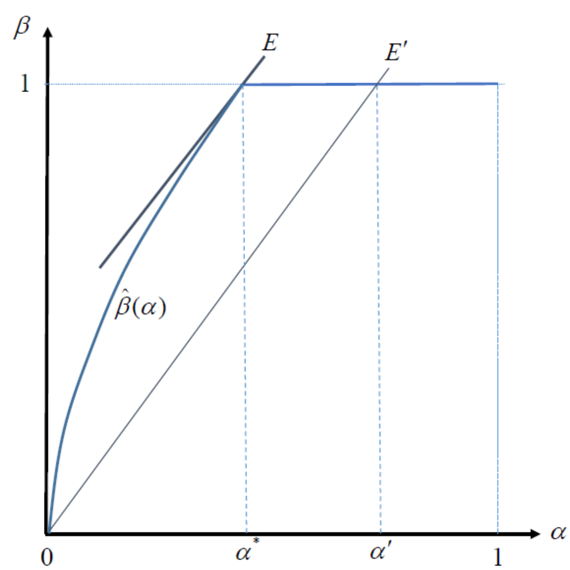

Fig. 6c. Cross and re-examination: corner solution with $\widehat{\beta}(\alpha^*) = 1$

The figures 6a-6c illustrate three possibilities. In all three cases, the equilibrium examination outcome under cross and re-examination is the binary signal with type 1 error denoted by α^*. In 6a, α^* is interior and solves

$$\hat{\beta}'(\alpha^*) = \frac{k_P(1-\theta)}{k_D\theta(1-e_D^{ae})}. \tag{35}$$

The cross-examination strategy that would have been chosen absent the right of re-examination is defined by α'. The slope of the corresponding straight line E' (recall condition (33)) is smaller than the slope $\hat{\beta}'(\alpha^*)$ of line E because the defendant investigates more in the procedure allowing re-examination, i.e., e_D^{ae} is larger.

Figure 6b illustrates a corner solution with $\alpha^* = 0$ and $\hat{\beta}(\alpha^*) > 0$. In this case

$$\hat{\beta}'(\alpha) < \frac{k_P(1-\theta)}{k_D\theta(1-e_D^{ae})} \tag{36}$$

for all $\alpha > 0$. Such an equilibrium may arise when the supports of f_n and f_m do not coincide. Finally, Figure 6c illustrates a different kind of corner solution with $\alpha^* > 0$ where α^* is the smallest type 1 error satsifying $\hat{\beta}(\alpha^*) = 1$. Now the inequality (36) holds only if $\alpha > \alpha^*$. Notice that a type-m counsel would gain nothing in refining the signal defined by α' but that a type-n counsel would want to do so, as this would strictly benefit the type-n plaintiff.

5.4 Quality of Decision Making

Using the same approach as before, the probability of correct adjudication is $\overline{u}_A = 1 - p + e_P\Lambda$ where Λ is the value of communication, including examination when it arises.

When allowing cross-examination yields the pd-equilibrium, the plaintiff's investigation effort is e_P^{pd} and the value of communication is Λ^{pd} as previously defined. When the outcome is an ae-equilibrium, the plaintiff's effort is e_P^{ae} and the value of communication is

$$\Lambda^{ae} = \Lambda^{pd} + [k_P(1-\theta)(1-\alpha) - k_D\theta(1-e_D^{ae})(1-\hat{\beta}(\alpha))], \tag{37}$$

where e_D^{ae} and α depend on whether only cross-examination is allowed or both cross and re-examination.

Cross-examination only. Substituting from the equilibrium condition (33), the value of communication reduces to

$$\Lambda^{ae} = \Lambda^{pd} + [k_P(1-\theta) - k_D\theta(1-e_D^{ae})]. \tag{38}$$

The expression in brackets is positive because condition (27) in lemma 4 must hold. The expression in (38) is identical to its counterpart in (21) for the case where cross-examination is not allowed, except for the defendant's investigation effort. Thus, allowing cross-examination would have no effect on the value of communication if it were not for the negative effect on the investigation effort of the party benefitting from cross-examination.

To put this into perspective, recall that in Kamenica and Gentzkow's (2011) motivating example, a prosecutor structures his arguments so as to persuade the judge to render guilty verdicts. The prosecutor can successfully do this. Guilty verdicts are reached more often than if the judge ruled on the basis of his prior. What is not emphasized is that the judge's expected utility (similar to that of our arbitrator) is unaffected by Bayesian persuasion. Transposed to the present setting, as in our motivating example of section 2, the interpretation is that cross-examination would neither improve nor deteriorate the value of communication, were it not for the disincentive effects on the defendant's gathering of evidence.

Proposition 5 *When $\theta < \theta_b$ and condition I holds, allowing cross-examination reduces the value of communication, $\Lambda < \Lambda^{ad}$. It reduces the quality of adjudication whenever $e_P \leq e_P^{ad}$. A sufficient condition for the latter is that the examination set is sufficiently rich.*

That $\Lambda < \Lambda^{ad}$ follows from previous results when the opportunity of cross-examination results in the *pd*-equilibrium, which also implies $e_P < e_P^{ad}$. When $\theta < \theta_a$ and the outcome is an *ae*-equilibrium, $\Lambda^{ae} < \Lambda^{ad}$ because $e_D^{ae} < e_D^{ad}$. On the other hand, cross-examination does not necessarily reduce the plaintiff's investigation effort. Although the plaintiff now faces the threat of cross-examination, which by itself reduces his incentives to investigate, it also reduces the risk that the defendant presents counterevidence because the defendant investigates less. The first effect dominates when cross-examination is sufficiently powerful in detecting the withholding of evidence.

An example is the case in figure 4b. Because $\hat{\beta}(\alpha^*) = 1$, the defendant does not investigate. The plaintiff's investigation effort therefore solves

$$C'(e_P^{ae}) = (1-\theta)P(x_1)(1-\alpha^*) + \theta P(y_1)$$

Comparing with (12), clearly $e_P^{ae} < e_P^{ad}$. By continuity, because of the positive type 1 error, this will also be the case for $\hat{\beta}(\alpha^*) < 1$ and large.

Cross and re-examination. The following is an immediate consequence of proposition 4.

Corollary 1 *In an ae-equilibrium, when re-examination is allowed, the value of communication, as defined in (37), is maximized conditional on the equilibrium investigation effort of the defendant.*

The result is an instance of the role of competition in increasing information revelation, under appropriate conditions.[19] The outcome is then the same as if examination were conducted in a nonpartisan manner; for example, by the adjudicator herself, assuming she had access to the same examination set and were equally skillful. For instance, the adjudicator could herself ask the "one question too many".

Proposition 6 *When re-examination is allowed:*
(i) Compared to cross-examination only, the value of communication is increased. The plaintiff's investigation effort may or may not increase.
(ii) Compared to no examination opportunity, the defendant investigates less; either the plaintiff submits evidence less often, $e_P < e_P^{ad}$, or the communication phase is less informative, $\Lambda < \Lambda^{ad}$, or both.

The first part of (i) follows from corollary 1 and the fact that the defendant's investigation effort is (weakly) increased, as shown in proposition 4.

[19] For the Bayesian persuasion setting, Gentzkow and Kamenica (2017b) show that the equilibrium outcome is 'maximally informative' when the senders' preferences are strictly zero sum, the information environment is 'Blackwell connected', and senders have access to the same set of signals (their proposition 6). These conditions are satisfied in the present setting.

The defendant's effort remains unchanged only when $\beta = 1$ in both procedures so that $e_D = 0$. In this case, the type 1 error is smaller when re-examination is allowed; hence, the value of communication is increased. Moreover, the plaintiff investigates more, so that the quality of adjudication unambiguously improves. In the general case, however, one cannot guarantee that e_P always increases.[20]

When the comparison is with respect to no examination opportunity, both e_P and Λ may decrease. An example is when the outcome is the *pd*-equilibrium. In an *ae* outcome, the possibility that communication deteriorates is due to the fact that the defendant investigates less, without this being compensated by the information revealed through examination. More generally, there is a trade-off between e_P and Λ. If communication is more informative, then the plaintiff investigates less.

Numerical example. How the different procedures compare in terms of the quality of adjudication depends on many factors, e.g., the supply elasticity of investigation effort, the relative precision of the different pieces of evidence, and the characteristics of the examination set. The numerical example in Table 2 focuses on the latter. We consider two examination sets as defined by the upper boundary $\widehat{\beta}(\alpha)$. In one case $\widehat{\beta}_I(\alpha) = \alpha^{1-\gamma}$, in the other $\widehat{\beta}_{II}(\alpha) = 1 - (1-\alpha)^{\frac{1}{1-\gamma}}$ where $\gamma \in (0,1)$. In each case, a larger γ means a more informative examination set. We take the supply of investigation effort to be relatively inelastic, so that the parties' gathering of evidence does not vary too much across the situations considered.[21]

To interpret the results, consider first the situation where the arbitrator has direct access to the evidence. With probability $1-\theta$ she observes only x and with probability θ she observes both x and y. The probability of correct adjudication then equals 0.8. Across the different cases considered in table 2, the probability of correct adjudication varies relatively little, ranging from

[20] Allowing re-examination reduces both the type 1 error and the power of the test, which increases the plaintiff's incentive to investigate, everything else constant. However, depending on the investigation cost function, it may be that the defendant overcompensates through his own investigation effort.

[21] The equilibrium investigation effort of party j satisfies $C'(e_j) = b_j$ where b_j is shorthand for the marginal benefit. In the specification used for Table 2, the supply elasticity $(b_j/e_j)de_j/db_j$ equals 0.25.

0.70 to 0.74.

Table 3
Procedures and examination sets

	e_P	e_D	α	β	Λ	$\frac{1}{2}+e_P\Lambda$
No examination	.857	.520	–	–	.276	.737
Examination set I						
Cross-examination only						
$\gamma = .8$	.821	.363	.239	.751	.268	.720
$\gamma = .9$	.819	.294	.314	.891	.265	.714
Cross & re-examination						
$\gamma = .8$	.848	.439	.027	.487	.283	.740
$\gamma = .9$	.850	.468	.021	.679	.288	.744
Examination set II						
Cross-examination only						
$\gamma = .8$	.823	.372	.227	.725	.269	.721
$\gamma = .9$	.791	.120	.438	.997	.256	.702
Cross & re-examination						
$\gamma = .8$	.847	.476	.065	.286	.275	.733
$\gamma = .9$	.835	.376	.119	.846	.279	.733

Notes: for set I, $\widehat{\beta}_I(\alpha) = \alpha^{1-\gamma}$; for set II, $\widehat{\beta}_{II}(\alpha) = 1-(1-\alpha)^{\frac{1}{1-\gamma}}$; $C(e_j) = e_j^5/5, j \in \{P, D\}$; $\theta = .5$, $p = .5$, $P(x_i|\, \omega_i) = .7$, $P(y_i|\, \omega_i) = .9$, $i \in \{0,1\}$.

Under system $\widehat{\beta}_I$, when only cross-examination is allowed, both the value of communication and the quality of adjudication deteriorate compared to the no-examination benchmark. The deterioration is worse when the examination set is more informative. When re-examination is also allowed, both indicators improve and are in fact better than in the no-examination benchmark. Under system $\widehat{\beta}_{II}$, compared to the benchmark, the negative effects of allowing cross-examination only are at least as strong. Moreover, the combination of cross and re-examination now does not do better than the benchmark with respect to the quality of adjudication. When $\gamma = 0.8$, the value of communication is smaller than in the benchmark. When $\gamma = 0.9$, the value of communication is greater than in the benchmark, but the quality of adjudication is still smaller because of the plaintiff's reduced investigation effort.

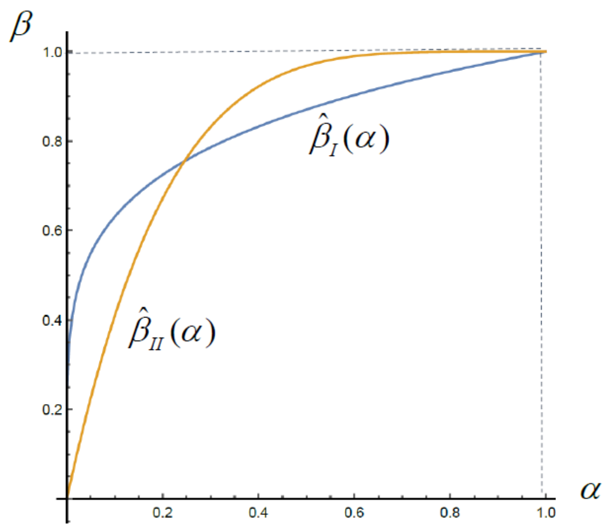

Fig. 7. Low vs high type 1 error orientation

Figure 7 compares the systems $\widehat{\beta}_I$ and $\widehat{\beta}_{II}$ when $\gamma = 0.8$. Because the curves intersect, the examination sets cannot be ranked in terms of one being more informative than the other.[22] However, $\widehat{\beta}_I$ performs better than $\widehat{\beta}_{II}$ at small levels of the type 1 error, which is where the equilibrium lies under cross together with re-examination. Loosely speaking, the first system is relatively more discriminating at small type 1 errors. For both examination sets, compared to the benchmark, the effect on the plaintiff's investigation effort is roughly the same but the defendant's investigation effort decreases less in case II. Nevertheless, decision making fares worse in case II (for the same value of γ) because of the larger type 1 and 2 errors. In other words, deceitful parties are detected less often and truthful parties more often appear to be liars.

[22] A standard measure of the discriminatory power of dichotomic tests is the area under the curve (AUC). In figure 6, the AUC under both curves equals 0.83. This would be considered relatively 'good' tests; see Hosmer and Lemeshow (2000).

6 Conclusion

Posner (1999, p. 1543) remarks that: "A principal social value of the right of cross-examination is deterrent: the threat of cross-examination deters some witnesses from testifying at all and others from giving false or misleading evidence." We studied a situation where cross-examination does indeed reduce the probability of testimony. However, we find that cross-examination can improve decision making only if it does not deter too much, both with respect to the party facing the threat of cross-examination and the one who stands to benefit from it.

We assumed a circumscribed pool of hard evidence pertaining to the fact at issue. The parties have relatively little discretion in this respect. They choose how much to invest in gathering evidence and what portion they will report. By contrast, a cross-examiner has much latitude in framing the test to which the opponent will be subjected. Being a partisan, the cross-examiner seeks to persuade that the opponent's report is deceitful or misleading. It suffices to raises just enough doubt that the report does not contain the whole truth. We find that uncontrolled cross-examination is detrimental to the quality of decision-making. Next, we examined a form of controlled examination by allowing the cross-examined to be re-examined by his own advocate. When the cross and re-examiner are equally skillful, this indeed corrects the excesses of cross-examination and is tantamount to what would arise under nonpartisan examination. However, whether decision making is improved compared to a procedure with no opportunity of examination at all is in general ambiguous. Although communication is optimized given the information available to the parties, both parties will be less likely to acquire relevant information. How this affects decision-making depends, in particular, on the trade-off that examination possibilities allow between 'false positives' and 'false negatives'.

We discussed the merits of examination solely with respect to the quality of decision-making. It may be that society is concerned both with decision-making and the parties' costs. The opportunity of examination typically reduces both parties' expenditure. Thus, there will be situations where decision-making is improved and costs are simultaneously reduced. Moreover, even when decision-making is not improved, society may still be better off given the weight accorded to procedural costs (see for instance Sobel, 1985).

In our setting, only one party found it useful to cross-examine the opponent. This followed mechanically from the simplifying assumption that the potential evidence consists of at most two pieces of information. The party with the burden of proof then sometimes submitted incomplete evidence, which the other party attempted to rebut by disclosing countervailing evidence or through cross-examination. Should that party disclose evidence, its report may itself be misleading if the potential evidence consists of more than two pieces. By relaxing the assumption on the structure of evidence, the analysis can be extended to bilateral cross-examination.

As a final remark, an extension of our analysis would be to relax the assumption that the cross-examiner and re-examiner (or a neutral examiner allowed to intervene, such as the arbitrator herself) are equally skillful. As a practical matter, this may be an important consideration. To illustrate, in our setting the procedure with cross-examination only is equivalent to one also allowing re-examination but with a totally unimaginative or ineffective re-examiner. A less extreme case would be a re-examiner who can only choose within a set of signals coarser than those available to the cross-examiner. The interpretation is that the re-examiner has limited ability to refine the cross-examiner's tests. Compared to cross-examination only and to cross and re-examination, one would expect the equilibrium outcome to be somewhere in between.

Appendix

Proof of Lemma 1. We show that $e_P^{ad} > e_D^{ad}$. Given $C'' > 0$, the claim is equivalent to $C'(e_P^{ad}) > C'(e_D^{ad})$ and therefore, using (12) and (13), to

$$(1-\theta)P(x_1) + \theta[P(y_1) + P(x_1, y_0)(1 - e_D^{ad})] > e_P^{ad}\theta P(x_1, y_0).$$

The left-hand side is decreasing in e_D^{ad} and the right-hand side increasing in e_P^{ad}, so it suffices that the inequality holds at $e_P^{ad} = e_D^{ad} = 1$, i.e.,

$$(1-\theta)P(x_1) + \theta[P(y_1) - P(x_1, y_0)] > 0. \tag{39}$$

The expression inside the brackets can be rewritten as

$$P(x_1, y_1) + P(x_0, y_1) - P(x_1, y_0)$$

Using (5),

$$\begin{aligned} P(x_1, y_1) - P(x_1, y_0) &= P(x_1, \omega_1)P(\ y_1 \mid \omega_1\ ) + P(x_1, \omega_0)P(\ y_1 \mid \omega_0\ ) \\ &\quad - [P(x_1, \omega_1)P(\ y_0 \mid \omega_1\ ) + P(x_1, \omega_0)P(\ y_0 \mid \omega_0\ ) \\ &= P(x_1, w_1)[2P(\ y_1 \mid \omega_1\ ) - 1] \\ &\quad - P(x_1, w_0)[2P(\ y_0 \mid \omega_0\ ) - 1] \end{aligned}$$

By assumption 1, $P(y_1 \mid w_1) \geq P(y_0 \mid w_0) > \frac{1}{2}$ and $P(x_1, w_1) > P(x_1, w_0)$, where the latter is equivalent to $P(\omega_1 \mid x_1) > \frac{1}{2}$. Hence, the expression is positive.■

Proof of Lemma 2. From Bayes' rule and given the communication strategies (6) and (7),

$$\mu((x_1, \emptyset), \emptyset; e_D) = \frac{\{(1-\theta)P(x_1, \omega_1) + \theta P(x_1, y_0, \omega_1)(1-e_D)\}e_P}{\{(1-\theta)P(x_1) + \theta P(x_1, y_0)(1-e_D))\}e_P}. \tag{40}$$

Therefore, $\mu((x_1, \emptyset), \emptyset; e_D) > \frac{1}{2}$ is equivalent to

$$\begin{aligned} &(1-\theta)P(x_1, \omega_1) + \theta P(x_1, y_0, \omega_1)(1-e_D) \\ &> (1-\theta)P(x_1, \omega_0) + \theta P(x_1, y_0, \omega_0)(1-e_D), \end{aligned} \tag{41}$$

which is equivalent to condition (14) with k_P and k_D as defined. The rest of the proof follows trivially.■

Proof of Proposition 1. We first show that $\mu(\emptyset, \emptyset) < \frac{1}{2}$ under both the *pd* and *ad* strategy profiles. Applying Bayes' rule,

$$\mu(\emptyset,\emptyset)=\frac{p\,\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_1)}{\Pr(r_P=\emptyset,r_D=\emptyset)}$$

so that $\mu(\emptyset,\emptyset)<\frac{1}{2}$ if

$$\frac{\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_0)}{\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_1)}>\frac{p}{1-p}. \tag{42}$$

Because $p\leq\frac{1}{2}$, the inequality (42) holds if it does for $p=\frac{1}{2}$, equivalently if

$$\xi\equiv\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_0)-\Pr(r_P=\emptyset,r_D=\emptyset\mid\omega_1)>0.$$

In the *ad*-equilibrium, given the communication strategies,

$$\begin{aligned}\Pr(\ r_P=\emptyset,r_D=\emptyset\mid\omega_i\ )=&\\ &(1-\theta)\{1-e_P+P(\ x_0\mid\omega_i\ )e_P\}\\ +\theta\{P(\ x_0,y_0\mid\omega_i\ )(1-e_D)+P(\ x_1,y_0\mid\omega_i\ )(1-e_P)(1-e_D)+P(\ y_1\mid\omega_i\ )(1-e_P)\}&\end{aligned}$$

Straightforward transformations then yield

$$\begin{aligned}\xi\ =\ & e_P\{(1-\theta)[P(x_1\mid\omega_1)-P(x_1\mid\omega_0)]-\theta[P(x_1,y_0\mid\omega_0)-P(x_1,y_0\mid\omega_1)](1-e_D)\}\\ &+\theta[P(\ y_1\mid\omega_1\ )-P(\ y_1\mid\omega_0\ )](e_P-e_D)\end{aligned}$$

The expression in the curly brackets is positive by condition (14) of lemma 2; the expression on the second line is also positive because $e_P>e_D$ by lemma 1. For the *pd*-equilibrium, set $e_D=0$ in the above expression. After some transformations, this yields

$$\xi=e_P\{(1-\theta)[P(x_1\mid\omega_1)-P(x_1\mid\omega_0)]+\theta[P(x_0,y_0\mid\omega_0)-P(x_0,y_0\mid\omega_1)]\}$$

which is obviously positive.

We show next that at least one of the *pd* or *ad* equilibria exists.

Case $\theta<\theta_a$. By Lemma 2, $\mu((x_1,\emptyset),\emptyset)>\frac{1}{2}$ and therefore $d((x_1,\emptyset),\emptyset)=1$ irrespective of e_D. Hence the plaintiff strictly gains by submitting $(x_1,\emptyset)$.

From Lemma 2, the unique equilibrium is then the ad-equilibrium with $e_P > e_D > 0$ solving (12) and (13).

Case $\theta \geq \theta_a$. By Lemma 2, $\mu((x_1, \emptyset), \emptyset) \leq \frac{1}{2}$ if $e_D = 0$. Therefore the pd-equilibrium with $e_P > e_D = 0$ exists. If the ad investment game has a solution $e_D > \varphi(\theta)$, then an ad-equilibrium also exists. We conclude by showing that this cannot arise for θ sufficiently large. Because $C'(1) \geq 1$, the conditions (12) and (13) imply $e_D < \bar{e} < 1$ where $\bar{e}$ solves $C'(e) = P(x_1, y_0)$. Because $\varphi(\theta)$ is strictly increasing and $\varphi(1) = 1$, it follows that $e_D^{ad}(\theta) < \bar{e} \leq \varphi(\theta)$ for all $\theta \geq \varphi^{-1}(\bar{e})$.■

Proof of Proposition 2. That $e_P^{ad} > e_P^{pd}$ follows trivially from the first-order conditions (12) and (10), given that $C'' > 0$. In the pd-equilibrium, $\Pr(d = 1 \mid \omega_i) = e_P^{pd} \theta P(y_1 \mid \omega_i)$. Substituting in (17) then yields (18) with Λ^{pd} as defined. In the ad-equilibrium,

$$\Pr\left(d = 1 \mid \omega_i\right) = e_P^{ad}\{(1-\theta)P(x_1 \mid \omega_i) + \theta[P(y_1 \mid \omega_i) + P(x_1, y_0 \mid \omega_i)(1-e_D)]\}.$$

Substituting in (17) then yields Λ^{ad} as defined.■

Proof of Lemma 3. The proof of the first part of the claim is similar to the argument used for lemma 1. That $e_D^{ae} < e_D^{ad}$ follows from the comparative statics of system (25)-(26) with respect to α and β.■

Proof of Lemma 4. The proof is similar to that of lemma 2 and is omitted.

Proof of Proposition 3. To complete the discussion in the text, we show that an ae-equilibrium exists when $\theta < \theta_a$. In system (25)-(26), write $\beta = \widehat{\beta}(\alpha)$ and let $e_D^{ae}(\alpha)$ denote the solution with respect to the defendant's investigation effort. It is easily verified that $e_D^{ae}(\alpha)$ is strictly decreasing. Moreover, $e_D^{ae}(1) = 0$ because $\widehat{\beta}(1) = 1$ and $0 < e_D^{ae}(0) \leq e_D^{ad}$ because either $e_D^{ae}(0) = e_D^{ad}$ when $\widehat{\beta}(0) = 0$ or $0 < e_D^{ae}(0) < e_D^{ad}$ when $\widehat{\beta}(0) > 0$.

Define

$$h(\alpha) \equiv 1 - \frac{\alpha}{\hat{\beta}(\alpha)} \frac{k_P(1-\theta)}{k_D \theta}. \tag{43}$$

The function is strictly decreasing. If $\widehat{\beta}(0) > 0$, then $h(0) = 1$. If $\widehat{\beta}(0) = 0$, we use l'Hôpital's rule and set

$$h(0) = 1 - \frac{k_P(1-\theta)}{k_D\theta} \lim_{\alpha\to 0^+}\left(\frac{1}{\hat{\beta}'(\alpha)}\right) = 1 - \frac{k_P(1-\theta)}{k_D\theta\hat{\beta}'(0^+)}$$

Moreover,

$$h(1) = 1 - \frac{k_P(1-\theta)}{k_D\theta} < 0,$$

where the inequality follows from $\theta < \theta_a$. Condition (33) is equivalent to $h(\alpha) = e_D^{ae}$. An ae-equilibrium with $\alpha \in (0,1)$ therefore exists if $t(\alpha) \equiv h(\alpha) - e_D^{ae}(\alpha) = 0$ has an interior solution.

The existence of such a solution follows from continuity and the fact that $t(0) > 0 > t(1)$. When $\widehat{\beta}(0) = 0$, condition I is satisfied if and only if there exists $\alpha > 0$ such that $h(\alpha) \geq e_D^{ad}$. Because $h(\alpha)$ is strictly decreasing, $h(0) > e_D^{ad} = e_D^{ae}(0)$ and therefore $t(0) = h(0) - e_D^{ae}(0) > 0$. Moreover, $t(1) = h(1) - e_D^{ae}(1) = r(1) < 0$. When $\widehat{\beta}(0) > 0$, condition I is trivially satisfied at $\alpha = 0$. We now have $t(0) = 1 - e_D^{ae}(0) > 1 - e_D^{ad} > 0$.■

Proof of Proposition 4. We show that the cross-examiner's problem, under the constraints (32) and (34), is equivalent to maximizing

$$g(\alpha) \equiv \hat{\beta}(\alpha)k_D\theta(1 - e_D^{ae}) - \alpha k_P(1-\theta) \tag{44}$$

with respect to α. First, if $\psi(z)$ satisfies (34), then

$$\left(\int_0^1 \psi(z)f_n(z)\,dz\right)k_P(1-\theta) \leq \left(\int_0^1 \psi(z)f_m(z)\,dz\right)k_D\theta(1-e_D^{ae}),$$

which is equivalent to (32), hence that constraint is redundant. Secondly, under (34), it is clear that the cross-examiner's objective is maximized by setting $\psi(z) = 1$ whenever

$$f_n(z)k_P(1-\theta) \leq f_m(z)k_D\theta(1-e_D^{ae}). \tag{45}$$

By assumption 2 (and Neyman-Pearson's lemma), this means that $\beta = \widehat{\beta}(\alpha^*)$ where $\alpha^* = \int_0^{z^*} f_n(z)\,dz$ and z^* is the largest z satisfying (45).

Recall that z_n and z_m are the upper bounds of the support of f_n and

f_m respectively. To satisfy (45), we must have $z^* \leq z_m$. Therefore, we can restrict attention to $\alpha^* \in [0, \alpha_m]$ where $\alpha_m \equiv \int_0^{z_m} f_n(z)\, dz$. Now, $g(\alpha)$ is maximized by some $\alpha \leq \alpha_m$ because $\widehat{\beta}'(\alpha) = 0$ for $\alpha > \alpha_m$. By assumption 2, $g(\alpha)$ is strictly concave in $[0, \alpha_m]$ and therefore has a unique maximum. If the maximum is interior, $g'(\alpha^*) = 0$ and $z^* \in (z_n, z_m)$ solves (45) as an equality. If the maximum is a corner at $\alpha^* = 0$, then $z^* = z_n$ and (45) never holds for $z > z_n$, equivalently $g'(\alpha) < 0$ for $\alpha > 0$. If the maximum is the corner $\alpha^* = \alpha_m$, then strict concavity implies $g'(\alpha) > 0$ for $\alpha \in (0, \alpha_m)$, equivalently (45) holds as a strict inequality for $z < z_m$.■

Proof of Proposition 5. The proof follows from the argument in the text.

Proof of Proposition 6. We prove claim (ii). We show that $\Lambda \geq \Lambda^{ad}$ together with $e_P \geq e_P^{ad}$ is impossible. These inequalities cannot hold in the *pd*-equilibrium, so we focus on an *ae*-equilibrium. Let α^* be the equilibrium type 1 error when both cross and re-examination are allowed. Then $\Lambda \geq \Lambda^{ad}$ is equivalent to

$$k_P(1-\theta)(1-\alpha^*) - k_D\theta(1-e_D^{ae})(1-\hat{\beta}(\alpha^*)) \geq k_P(1-\theta) - k_D\theta(1-e_D^{ad})$$

or

$$\frac{\alpha^* k_P(1-\theta)}{k_D\theta} + (1-e_D^{ae})(1-\hat{\beta}(\alpha^*)) \leq 1 - e_D^{ad}, \tag{46}$$

while $e_P^{ae} \geq e_P^{ad}$ is equivalent to

$$\begin{aligned}(1-\theta)P(x_1) &+ \theta[P(y_1) + P(x_1, y_0)(1-e_D^{ad})] \\ &\leq (1-\theta)P(x_1)(1-\alpha^*) + \theta[P(y_1) + P(x_1, y_0)(1-\hat{\beta}(\alpha^*))(1-e_D^{ae})]\end{aligned}$$

or

$$-\frac{\alpha^* P(x_1)(1-\theta)}{P(x_1, y_0)\theta} + (1-e_D^{ae})(1-\hat{\beta}(\alpha^*)) \geq 1 - e_D^{ad}. \tag{47}$$

Both (46) and (47) cannot simultaneously hold.■

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