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Wage Dynamics and Peer Referrals

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Abstract

We present a flexible model of wage dynamics where information about job openings is transmitted through social networks. The model is based on Calvò-Armengol & Jackson (2004, 2007) and extends their results outside the stationary distribution, and under observed and unobserved heterogeneity. We present an empirical application using the British Household Panel Survey by exploiting direct information about individual's social networks. We find that having more employed friends leads to more job offers but to lower wages due to higher mismatch. We also find that non-relative friends are more helpful than relatives, and that women benefit relatively more from their male friends.

Keywords: Labour Market, Peer Referrals, Social Networks

JEL Classification: C33, J31, J46.

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1. Introduction

Many workforce characteristics (such as wages) are determined outside formal market structures. For example, it is estimated that between 18% and 45% of jobs are found using personal contacts (Pellizzari, 2010). A significant portion of wage inequality between different groups, and the persistence of this inequality, may be due to differences in the composition of social networks (Ioannides & Soetevent, 2006; Fontaine, 2008). Understanding non-market forces governing employment and wages has been a preoccupation for economists, going back to Rees (1966), Granovetter (1973, 1983) and Montgomery (1991, 1992).

In this paper, we present a flexible structural framework for analyzing how non-market institutions (e.g. peer referrals) affect wages. In particular, we study how labour market transitions are affected by an individual's friendship network. We show that having more *employed* friends leads to more job offers (particularly for women), but lower wages due to higher mismatch. We explore the heterogeneity of peer influence and find that non-relative friends are more helpful than relatives and that women benefit more from their male (employed) friends than form their female (employed) friends.

A particular feature of our approach is that it allows for a systematic dependence between the individuals' wages. For example, the positive impact of an individual's peers on his wage is likely to grow with the proportion of his peers who are employed. This may result from the fact that employed individuals have better information on the state of the labour market, or from the fact that unemployed individuals may be more reluctant to share private information about jobs.

Our contribution is twofold. First, we contribute to the theoretical literature on network effects in the labour market (e.g. Calvó-Armengol & Zenou (2005), Cahuc & Fontaine (2009) and Fontaine (2008)) by extending the results of Calvó-Armengol & Jackson (2004, 2007). Specifically, we show that a natural extension of their results holds 1) outside the stationary distribution, and 2) under observed and unobserved heterogeneity. Second, we present an empirical application using data from the British Household Panel Survey (BHPS) from 2000 to 2006, where we exploit direct information on individuals' friendship networks and the employment status of their friends.

We build on the important contribution of Calvó-Armengol & Jackson (2004, 2007) by extending their model to include observed heterogeneity (e.g. gender) and unobserved heterogeneity (i.e. random-effect model). In contrast with most of the literature,[1] our results also hold outside the stationary distribution. This is empirically important since

[1]See Ioannides & Loury (2004) for an extensive review of the literature on social networks and the labour market.

periods of interest often include short-term events such as recessions. In other words, we do not need to assume that the data is generated from the stationary distribution.

We find that the individuals' wages dynamic is *associated*. This implies that, conditional on the observables, the wages of any two individuals are positively correlated, across any point in time. We also show that, as time passes, this dependence is strict for any two socially connected individuals, and that the speed at which this dependence spreads can be expressed as a function of the social network. This allows us to describe the impact of a shock to an individual's wage on the overall distribution of wages, at any point in time.

We restrict our analysis to a time-invariant network. Although our theoretical model abstracts away from strategic network formation considerations, such as in Calvó-Armengol (2004) and Galeotti & Merlino (2014), our empirical analysis controls for the endogeneity of the network structure as in Qu & Lee (2015) and Hsieh & Lee (2014). Coherently with the literature (see Boucher & Fortin (2015)), we find little difference between the model allowing for an endogenous network and the model assuming that the network is exogenous.

Importantly, our theoretical framework allows to describe labour market transitions across potentially long periods of time. Wages are therefore correlated across individuals and time, alowing us to explore a rich variety of channels through which peer effects operate. Specifically, within a single coherent framework, we can separately identify the impact of having employed friends on the probability of receiving job offers, on the wage of such offers, as well as on the separation rate.

We present an empirical application using data from the BHPS from 2000 to 2006. We build on our theoretical framework and develop a non-linear panel dynamic spatial autoregressive (DSAR) model. An important feature of our model is that an individual's wage is not only dependent on his position in the network, but also on the employment status of the other individuals in the network. We model the dependence on the initial state using random effects, as in Wooldridge (2005).

We find that the number of employed friends an individual has at time t has a positive impact on the probability of receiving a job offer at time $t+1$. We find that this effect is much stronger for women. However, due to mismatch, the distribution of such offers is consistently lower leading to lower wages. This result is in line with findings of stronger peer-effects for women (Dieye & Fortin, 2014; Neumark & Postlewaite, 1998). Moreover, for women, having more employed friends tend to lower the separation rate.

We also explore the heterogeneity of peer influence. We find that recent (employed) friends have a stronger impact on the probability of receiving a job offer. We also find a higher mismatch for (employed) female friends and relatives. This is particularly true for women. In particular, we do not find any mismatch for women having male friends.

Our findings enrich the existing literature (e.g. Ibarra (1992), Campbell (1988), Hanson & Pratt (1991) and Marmaros & Sacerdote (2002)) that find that women's job networks are relatively "poor" as compared to men's (lower density and "quality") and that social networks contribute to the gender gap in wages and promotions. This is an indication that women's wage may be more dependent on their social connections (particularly with male workers) than men (Mota et al., 2016).

We contribute to the empirical literature on the effects of networks on the labour market (e.g. Ioannides & Soetevent (2006)). There are still relatively few empirical works looking at friendship networks. This is mostly due to lack of detailed data.[2] Existing studies use either information on close neighbourhoods (Bayer et al., 2009), or on co-workers' networks (Cingano & Rosolia, 2012; Dustmann et al., 2011; Åslund et al., 2014). Kramarz & Skans (2014) analyze the effect of strong family ties on young workers' success in finding jobs. Galeotti & Merlino (2014) use data on friends and relatives, although they do not observe their employment status.

We use the information provided by the BHPS, which contains direct information about individuals' friendship networks and the employment status of their friends. To our knowledge, Cappellari & Tatsiramos (2015) is the only paper exploiting this data in a similar fashion. They model the transition probability from unemployment to employment using fixed effect, random effect and instrumental variable strategies and find that having one more employed friend increases an individual's probability of leaving unemployment by 15%. They also find evidence of mismatch modeling the first wage equation obtained after unemployment[3]. We complement and extend their analysis by proposing a structural non-linear DSAR model in which friends' wages are correlated across time. In particular, our approach focus on labour market transitions, over many periods. In our context, individuals can receive job offers while employed, and are free to refuse offers with unattractive wages. This considerably enrich the scope of peers' influence on wage dynamics.

Our analysis also complements Arulampalam & Stewart (2009), who also use the BHPS in order to estimate the dynamics of wages and employment using a similar methodology. We enrich their findings by including the impact of the friendship network.

The remainder of the paper is organized as follows. In section 2, we present the microeconomic framework. In section 3, we present our structural econometric model.

[2]Substantial work has been done on the impacts of friendship networks of teenagers, using the Addhealth database. See `http://www.cpc.unc.edu/projects/addhealth`.

[3]They do not run separate analysis for men and women.

We conclude in section 4.

2. Wage Distributions with Network Effects

In this section, we present a non-linear model of wage dynamics with peer referrals. We show that, under natural assumptions, the individuals' wages are positively correlated, and that the dependence between the individuals' wages can be described by the social network. The model allows for many types of peers' influence. In particular, peers' wages can act through the layoff probability, the probability of job offers, as well as through the equilibrium wage. The model is also well behaved in the sense that it allows for simple comparative dynamics and is (weakly) ergodic.

We consider an economy composed of a finite set of individuals, N. Each individual $i \in N$ is characterized by a time-varying type $[\mathbf{x}_i^t, \varepsilon_i^t]$, where $\mathbf{x}_i^t$ is observed, but not ε_i^t. Typically, $\mathbf{x}_i^t$ will include socio-economic characteristics such as the individual's gender, level of education, and age. We assume that the $\boldsymbol{\varepsilon}^t$ are independent and identically distributed across time only, so ε_i^t and ε_j^t may be correlated for a given period of time t. Individuals interact in a time-invariant social network represented by the matrix $\mathbf{G}^s$, such that $g_{ij}^s = 1$ if i and j are linked, and $g_{ij}^s = 0$ otherwise. For instance, $\mathbf{G}^s$ may represent friendships (Galeotti & Merlino, 2014) or family ties (Kramarz & Skans, 2014). We denote by $N^s(i) = \{j \in N : g_{ij}^s = 1\}$ the set of n_i^s individuals linked to i (i.e. the set of i's peers). We also denote by $\rho^s(i,j)$ the shortest path between i and j. The shortest path is the minimum number of links needed to reach j from i in the network. If it is not possible to reach j from i, we let $\rho^s(i,j) = \infty$. If $\rho^s(i,j) < \infty$, we say that i and j are *socially connected*. At every period $t = 0, 1, ...$, each individual earns a wage $w_i^t \in W \equiv [b, \infty)$, where $b \geq 0$ is interpreted as social benefit.[4]

We assume that the evolution of wages can be described as follows:

$$\mathbf{w}^{t+1} = \varphi(\mathbf{w}^t, \mathbf{X}^t, \boldsymbol{\varepsilon}^t) \tag{1}$$

A simple example of (1) is a standard AR(1) process: $w_i^{t+1} = \rho w_i^t + \mathbf{x}_i^t \boldsymbol{\beta} + \varepsilon_i^t$. However, we are interested in a wider variety of economic situations, where individuals' wages can be correlated among themselves.

Since the $\boldsymbol{\varepsilon}^t$ are iid across time, the evolution of wages can be described by a non-homogeneous Markov process. Accordingly, it will sometimes be convenient to describe the distribution of wages using probability measures. We can denote the conditional

[4] We assume that wages include any non-monetary benefit or cost associated with an individual's job, so preferences are strictly increasing in wages.

probability of $A \in \mathcal{B}^n$ at $t+1$, given $\mathbf{w}^t$ and $\mathbf{X}^t$ as $P(\mathbf{w}^t, \mathbf{X}^t; A)$, where $\mathcal{B}^n$ is the Borel set on $W \subset \mathbb{R}^n$.[5] We also define inductively

$$P^{t+1}(\mathbf{w}^0, (\mathbf{X}^t); A) = \int_W P(\mathbf{w}, \mathbf{X}^t; A) P^t(\mathbf{w}^0, (\mathbf{X}^{t-1}); d\mathbf{w})$$

where we use the short-hand notation $(\mathbf{X}^t) \equiv (\mathbf{X}^0, ..., \mathbf{X}^t)$. If we further assume that $\mathbf{X}^t = \mathbf{X}$ for all t, the model reduces to a standard (homogeneous) Markov chain, as in Calvó-Armengol & Jackson (2004, 2007).

This model allows for a rich variety of interactions between individuals, as illustrated in Example 1.

Example 1. *Suppose that, at every period t, the economy is described by the following phases:*[6]

1. *Each individual is laid off with probability $\delta_i(\mathbf{w}^t_{-i}, \mathbf{w}^{t-1}, \mathbf{X}^{t-1}; \mathbf{G}) \in (0,1)$.*
2. *Each individual receives a job offer with probability $\gamma_i(\mathbf{w}^t_{-i}, \mathbf{w}^{t-1}, \mathbf{X}^{t-1}; \mathbf{G}) \in (0,1)$*
3. *If an offer is received, the offer follows a distribution $\Lambda_i(\mathbf{w}^t_{-i}, \mathbf{w}^{t-1}, \mathbf{X}^{t-1}; \mathbf{G})$*
4. *If an offer is received, and the offer is greater than the individual's current wage, the individual accepts the offer.*

In this setup, network effects can act through many channels. First, they can affect the separation rate δ_i. Second, it can affect the probability of receiving job offers γ_i, as displayed portrayed in Calvó-Armengol & Jackson (2004). Finally, it can also affect the negotiation of wages. For example, it can reflect the fact that jobs received through social connection exhibit higher mismatch, and therefore lower wages. (Cappellari & Tatsiramos, 2015) The distribution of job offers Λ_i could also reflect the fact that friends with highly paid jobs are more likely to "transfer" job offers for relatively highly paid jobs. Figure 1 illustrates a typical conditional cumulative wage distribution.

Note that the specification of the dynamics of wages in Equation 1 will generally impose some restrictions on individual rationality. In Example 1, for instance, we assume that individuals accept any job offer that pays a higher wage. This decision is clearly rational in the short run. However, it may be possible that accepting a highly paid job today reduces the prospect of finding an even better paying job in the future. In this case, the naive decision process described in Example 1 would not be rational in the long run. We abstract from these effects by assuming that an increase in wages cannot reduce an individual's future wage prospects.[7] Formally:

Assumption 1 (Monotonicity). *$\varphi(\mathbf{w}^t, \mathbf{X}^t, \boldsymbol{\varepsilon}^t)$ is increasing in all its arguments.*

The assumption that $\varphi(\mathbf{w}^t, \mathbf{X}^t, \boldsymbol{\varepsilon}^t)$ is increasing in $\mathbf{X}^t$ merely implies that variables in $\mathbf{X}$ should be thought "positive" for the individuals (e.g. level of Education). The assumption

[5] Note that P does not depend on t since the $\boldsymbol{\varepsilon}^t$ are independent and identically distributed.

[6] *For the sake of the argument, we expose the model using the conditional distributions, implicitly assuming that they are coherent with some joint distribution.*

[7] Note that this assumption imposes more than monotonicity with respect to an individual's wage, as it is also increasing in other individuals' wages. This assumption is also implicitly present in Calvó-Armengol & Jackson (2004) (lemma 8) and Calvó-Armengol & Jackson (2007) (lemma A.4).

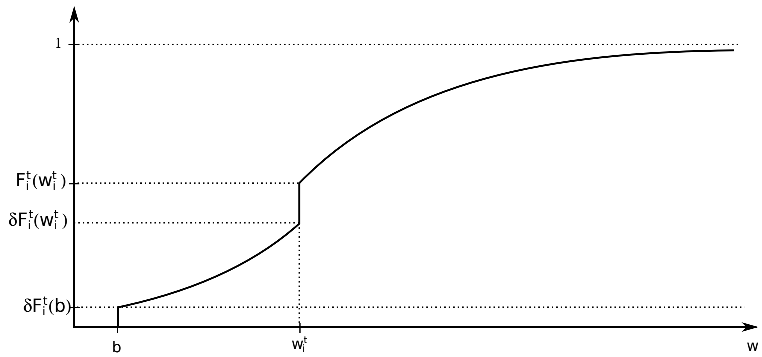

Figure 1: (Example 1) Fix w_i^t, and let F_i^t represent the cumulative distribution of offers for i at time t (so γ_i and Λ_i are implicitly embedded in F_i^t). The probability that i becomes unemployed is equal to the probability that he gets laid off and that he does not receive any job offer paying more than the welfare level, i.e. $\delta_i F_i^t(b)$. The "jump" observed at w_i^t is equal to the probability that the individual keeps his current job, which is equal to the probability that he does not get laid off, and that he receives an offer that is less than w_i^t, i.e. $(1-\delta_i)F_i^t(w_i^t)$.

that $\varphi(\mathbf{w}^t, \mathbf{X}^t, \boldsymbol{\varepsilon}^t)$ is increasing in $\mathbf{w}^t$ and $\boldsymbol{\varepsilon}^t$ is equivalent to the following: everything else equal, increasing wages today leads to a better distribution of wages tomorrow, in the sense of first-order stochastic dominance (FOSD).[8]

A powerful implication of Assumption 1 is that it allows for simple comparative dynamics of the model. Let $\succeq$ represent dominance in the sense of the FOSD, we have the following:

Proposition 1. *Suppose that Assumption 1 holds and let $\mathbf{w}^0 \succeq \tilde{\mathbf{w}}^0$ and $(\mathbf{X}^t) \geq (\tilde{\mathbf{X}}^t)$. Then $\mathbf{w}^t|(\mathbf{X}^{t-1}) \succeq \tilde{\mathbf{w}}^t|(\tilde{\mathbf{X}}^{t-1})$ for all t.*

Proposition 1 follows from standard results for the comparison of monotone stochastic processes (see the appendix for a proof and references).

Suppose that $\mathbf{X}^t$ represents individuals' levels of education. This implies that increasing the level of education of some individual i will have a positive impact on the entire distribution of wages. It also implies that the order between two wage dynamics will be preserved for all t.

Note that in principle, Proposition 1 could be applied to changes in the network structure. Suppose that $\mathbf{G}^s \supseteq \tilde{\mathbf{G}}^s$ implies that $\varphi_{\mathbf{G}^s}(\mathbf{w}, \mathbf{X}, \boldsymbol{\varepsilon}) \geq \varphi_{\tilde{\mathbf{G}}^s}(\mathbf{w}, \mathbf{X}, \boldsymbol{\varepsilon})$. That is, adding links to the social network has a positive impact on the wage distribution at a given time. Then, Proposition 1 implies that this dominance holds for all t. Such an assumption may represent situations where the information transmitted through the network is non-rival. For example, individuals may gain information from their peers about how to access government programs or evade taxes (Bellemare et al., 2012). The

[8]Recall that $\mathbf{w}$ is greater than $\mathbf{w}'$ in the sense of FOSD iff $\mathbb{E}u(\mathbf{w}) \geq \mathbb{E}u(\mathbf{w}')$ for any non-decreasing bounded function u. See Müller & Stoyan (2002), theorem 5.2.3 for a proof of this claim.

key is that when information is non-rival, everyone benefits from an individual having more links.

However, if the information transmitted through the network is rival, the addition of a link will usually be beneficial to some, but detrimental to others. For example, individuals may prefer to have many links since it increases the probability that they receive job offers. However, they prefer to be linked (all else equal) to individuals with relatively few links, as it increases the probability that such an individual will transmit an offer to them, and not to another peer. Increasing the number of links will therefore have ambiguous effects on the wage distribution (see for instance Calvó-Armengol & Jackson (2004)).

Under Assumption 1, an increase of some individual's wage has a non-negative impact on every individual's wage. However, the effect might not be strict and some individuals' wages may be independent. The dependence structure of $\mathbf{w}^t$ is affected by the shape of φ, as well as by the dependence structure of $\boldsymbol{\varepsilon}^t$. In the next section, we discuss the dependence structure of the wage distribution.

2.1. Dependence Structure

We first introduce our notion of positive dependence.

Definition 1 (Association). *Consider the random vector* $\mathbf{w}$. *We say that* $\mathbf{w}$ *is* ***associated*** *if*

$$Cov(a(\mathbf{w}), c(\mathbf{w})) \geq 0$$

for all non-decreasing functions a *and* c.

In particular, letting $a(\mathbf{w}) = w_i$ and $c(\mathbf{w}) = w_j$ for some $i, j \in N$, we have the following corollary: if $\mathbf{w}$ is associated, then $Cov(w_i, w_j) \geq 0$ for all $i, j \in N$. Association has been used in a very similar context by Calvó-Armengol & Jackson (2004, 2007). We assume the following:

Assumption 2 (Dependence Structure). *We assume that:*

1. $\boldsymbol{\varepsilon}^t$ *is associated for all* t, *and that*
2. *for any* $\mathbf{w}, \mathbf{X}$, $P(\mathbf{w}, \mathbf{X}; A) > 0$ *for all strictly positive* $A \in \mathcal{B}^n$.

In particular, Assumption 2.1 implies that unobserved shocks are positively correlated across individuals. Note that the case where the ε_i^t are independent is a special case. Assumption 2.2 is not particularly strong as any non-positive distribution can be closely approximated by a positive distribution. It simply implies that any realization of wages is possible. This assumption is convenient in order to simplify the exposition of the model.[9]

An important consequence of Assumption 2 (together with Assumption 1) is the following:

[9]See proofs of Propositions 3 and 4, in the appendix, for details.

Proposition 2 (Association). *Suppose that* $\mathbf{w}^0$ *is associated and that Assumptions 1 and 2.1 hold. Then,* $(\mathbf{w}^t)|(\mathbf{X}^{t-1})$ *is associated, for all* t.

This result follows directly from the literature on monotone stochastic processes (references are provided in the appendix). Recall that $(\mathbf{w}^t) = (\mathbf{w}^0, ..., \mathbf{w}^t)$, so Proposition 2 implies that association holds for any two individuals, across any points in time. In particular, it implies that wages are positively correlated (see Calvó-Armengol & Jackson (2004, 2007)).

However, Proposition 2 (like Proposition 1) includes two unwanted features. First, it only describes *weak* dependence, since any two independent variables are necessarily associated. Second, it depends on the initial state $\mathbf{w}^0$, which may be unknown in practice.

We first describe the dependence structure of the wage distribution. Specifically, we want to know which individuals' wages are dependent, and which are not. It will be useful to describe the dependence of the stochastic process using a network structure.[10] However, that network structure *may not* be the same as the social network structure $\mathbf{G}^s$. The dependence structure can be summarized as follows:

Definition 2 (Dependence Network). *There exists a network* $\mathbf{G}^d$*, called the "dependence network," which is the smallest network such that for all* $i \in N$*:*

$$w_i^{t+1}|\mathbf{w}_{-i}^{t+1}, \mathbf{w}^t, \mathbf{X}^t = w_i^{t+1}|\mathbf{w}_{N^d(i)}^{t+1}, \mathbf{w}_{N^d(i)\cup\{i\}}^t, \mathbf{x}_i^t$$

Definition 2 highlights two important features of the model. First, the dependence network characterizes the dependence structure of the wage distribution at any point in time. Note that this is done without any loss of generality, since $\mathbf{G}^d$ can be the complete network. Also note that $\mathbf{G}^d$ is stable through time, since the $\boldsymbol{\varepsilon}^t$ are independent and identically distributed. Second, the model is limited to endogenous interactions: an individual is not affected by his peers' types, only by their wages.[11]

Note that even if $\mathbf{G}^s$ and $\mathbf{G}^d$ need not be related, the dependence network will often turn out be a function of the social network. Example 2 highlights the differences between $\mathbf{G}^s$ and $\mathbf{G}^d$.

Example 2 (Example 1 continued). *In order to clearly expose the differences between* $\mathbf{G}^s$ *and* $\mathbf{G}^d$*, consider again the framework of Example 1. For simplicity, we further assume that for all* $i \in N$*,* $\delta_i(\mathbf{w}^{t-1}, \mathbf{X}^{t-1}; \mathbf{G}) = \delta$ *and that* $\Lambda_i(\mathbf{w}^{t-1}, \mathbf{X}^{t-1}; \mathbf{G}) = \Lambda(w_i^{t-1}, \mathbf{x}_i^{t-1})$*. Then, peers can only affect an individual's wage through* $\gamma_i(\mathbf{w}^{t-1}, \mathbf{X}^{t-1}; \mathbf{G})$*. Lets further assume that:*

$$\gamma_i(\mathbf{w}^{t-1}, \mathbf{X}^{t-1}; \mathbf{G}) = \Gamma(\alpha_0 + \mathbf{x}_i\boldsymbol{\alpha}_1 + \alpha_2 \sum_{j \neq i} \mathbb{1}[\rho(i,j) \leq \kappa] w_j^{t-1})$$

[10] Formally, $\{w_i^t, i = 1, ..., n, t = 0, ..., T\}$ forms a Markov random field for any $T > 0$.

[11] Note that this is also done without loss of generality since one can always define $\mathbf{x}_i$ as $[\hat{\mathbf{x}}_i, \hat{\mathbf{X}}_{N^d(i)}]$ for some initial matrix $\hat{\mathbf{X}}$. In this case, however, any change in $\mathbf{G}^d$ leads to a change in $\mathbf{X}$, which may be important for the comparative dynamics of the model.

where $\Gamma(\cdot) \in (0, 1)$.

Here, the probability of receiving an offer depends on the individual's characteristics $\mathbf{x}_i$*, as well as on the wage of the individuals located within a distance of* κ *in the social network. If* $\kappa = 1$*, then the probability of receiving a job offer only depends on direct friends, and therefore* $\mathbf{G}^d = \mathbf{G}^s$*, however if* $\kappa > 1$*, the probability of receiving a job offer also depends on indirect friends. Then, in this particular example,* $\mathbf{G}^d_{ij} = 1$ *iff* $\rho(i, j) \leq \kappa$.

An advantage of using Definition 2 is that the distance in the dependence network allows us to describe how fast shocks (e.g. information) spread in the economy. Formally:

Proposition 3. *Under Assumption 2.2,*

$$(w_i^t \perp w_j^t)|\mathbf{w}^0, (\mathbf{X}^{t-1}) \quad \textit{iff} \quad \rho^d(i, j) > t$$

See the appendix for a proof. Remark that this last proposition only depends on assumptions 2.2 and does not rely, for instance, on monotonicity. Since the dependence network characterizes the dependence of $\mathbf{w}^t$, the shortest path in the dependence network allows us to describe how many periods are needed in order for a shock to spread from one individual to another. Note that if the dependence network is such that $\rho^d(i, j) < \infty$ iff $\rho^s(i, j) < \infty$ for all $i \neq j$, Propositions 2 and 3 imply that the wages of any socially connected individuals are *strictly* associated after some finite amount of time.[12]

Now, note that Proposition 3 suffers from the same limitation as Propositions 1 and 2: the dependence on the initial state, $\mathbf{w}^0$. We study the asymptotic behavior of the model in the next section.

2.2. Asymptotic Behavior

We now provide sufficient conditions for the dependence on $\mathbf{w}^0$ to vanish asymptotically. We assume the following:

Assumption 3 (Layoff Probability). *There exists* $\underline{\delta} \in (0, 1)$ *such that* $\underline{\delta} \leq P(\mathbf{w}, \mathbf{X}; \mathbf{b})$ *for all* $\mathbf{w}, \mathbf{X}$.

This assumption is quite standard and assumes a positive separation rate for every individual. Technically, it ensures the recurrence of the stochastic process at state $\mathbf{b}$.

Let us define the *total variation norm* as $\|P\| = 2 \sup_{A \subseteq W} |P(A)|$. Then, the next proposition follows. (See the appendix for a proof.)

Proposition 4. *Under Assumptions 2.2 and 3, and for any* $\mathbf{w}^0$ *and* $\tilde{\mathbf{w}}^0$*:*

$$\|P^t(\tilde{\mathbf{w}}^0, (\mathbf{X}^{t-1}); \cdot) - P^t(\mathbf{w}^0, (\mathbf{X}^{t-1}); \cdot)\| \to 0$$

[12] This generalizes Calvó-Armengol & Jackson (2004) (Proposition 1) and Calvó-Armengol & Jackson (2007) (Theorem 1), where they present results for the stationary distribution of a homogeneous Markov chain.

as $t \to \infty$. Moreover, if $\mathbf{X}^t = \mathbf{X}$ for all t, there exists a unique probability measure π such that for any $\mathbf{w}^0$:

$$\|P^t(\mathbf{w}^0, \mathbf{X}; \cdot) - \pi\| \to 0$$

as $t \to \infty$.

When $\mathbf{X}^t$ is time dependent, the model may not have a stationary distribution. However, as $t \to \infty$, the dependence on the initial state vanishes. When $\mathbf{X}^t = \mathbf{X}$ for all t, the model *does* have a stationary distribution, π. Finally, note that proposition 4 only depends on assumptions 2.2 and 3, and not on the monotonicity of stochastic process.

This completes the analysis of the framework. In the next section, we present our parametric assumptions.

3. A Structural Model of Wage Dynamics with Peer Referrals.

While the framework presented in the previous section is very general, we still need to impose additional restrictions, mostly due to the availability of the data (see next section for a precise description). In this section, we therefore present a structural model, based on Example 1. Let $E_i^t = \sum_{j \in N^s(i)} \mathbb{1}(w_j^t > b)$ denote the number of i's peers who are employed at time t, we assume that every period is characterized by the following phases.

1. Each individual is laid off with probability $\delta_i^{t+1} = \delta + \delta_{EF} E_i^t$.
2. Each individual receives a job offer with probability $\gamma_i^{t+1} = \gamma + \gamma_{EF} E_i^t$.
3. If an offer is received, the offer follows a distribution $\mathbf{\Lambda}(\mathbf{X}^t, \mathbf{w}^t)$, which we assume to be a log-normal distribution,

$$\ln \omega_i^{t+1} = \mathbf{x}_i^t \boldsymbol{\beta} + \lambda \ln(w_i^t) + \tau E_i^t + \varepsilon_i^t$$

   with $\lambda, \tau > 0$ and ε_i^t is normally distributed.
4. If an offer is received, and the offer is greater than the individual's current wage, the individual accepts the offer.

It is worth noting that E_i^t is a function of $\mathbf{G}^s$ and $\mathbf{w}^t$. Note also that, as in Example 1, the models allows for many sources of social interaction. Indeed, peers can affect the separation rate, the probability of receiving job offers, as well as the distribution of offers. We can write i's wage at $t+1$ as follows:

$$\begin{aligned}
\ln(w_i^{t+1}) \quad &= \quad (1 - D_{\delta_i^{t+1}}) D_{\gamma_i^{t+1}} \max\left\{0, \left(\mathbf{x}_i^t \boldsymbol{\beta} + (\lambda - 1)\ln(w_i^t) + \tau E_i^t + \varepsilon_i^t\right)\right\} \qquad (2) \\
&+ \quad D_{\delta_i^{t+1}} D_{\gamma_i^{t+1}} \max\left\{0, \left(\mathbf{x}_i^t \boldsymbol{\beta} + \lambda \ln(w_i^t) - \ln(b) + \tau E_i^t + \varepsilon_i^t\right)\right\} \\
&+ \quad (1 - D_{\delta_i^{t+1}}) \ln(w_i^t) + D_{\delta_i^{t+1}} \ln(b)
\end{aligned}$$

where D_p denotes the (Bernouilli distributed) random variable that takes a value of 1 with probability p, and 0 otherwise.

We obtain a dynamic nonlinear panel data model that we can estimate using actual data. Such a model needs some additional assumptions to deal with the initial condition issue. Specifically, we do not observe the initial state, but it is likely to be correlated with individuals' unobserved characteristics. To our knowledge, there is no transformation that controls for unobservable individual fixed effects in non-linear settings. We therefore use the estimation method proposed by Wooldridge (2005) and assume the following random effects model:[13]

$$\varepsilon_i^t = \alpha_i + u_i^t,$$

where $u_i^t \sim \mathcal{N}(0, \sigma_u)$.

We then model the random effects as a function of the initial conditions. Specifically:

$$\alpha_i = \theta \ln w_i^0 + \eta_i,$$

where $\eta_i \sim \mathcal{N}(0, \sigma_\eta)$ and w_i^0 is i's initial wage.

To provide more flexibility in the specification of the conditional distribution of the unobserved effect, we allow α_i to be correlated with the exogenous regressors over all the periods. To this end, we use the Mundlak specification of correlated random effects (CRE) (Mundlak, 1978):

$$\alpha_i = \theta \ln w_i^0 + \boldsymbol{z_i \rho} + \eta_i,$$

where $\boldsymbol{z}_i = (\bar{x}_{i1}, ..., \bar{x}_{iK}, \ln w_i^0 \times \bar{x}_{i1}, ..., \ln w_i^0 \times \bar{x}_{iK})$ is a vector of the means of the exogenous regressors over all the periods and their interaction with the initial state. The individual-specific random-effects specification implies that the correlation between $v_i^t = \eta_i + u_i^t$ in any two different periods will be the same: $= corr(v_i^t, v_i^s) = \sigma_\eta^2/(\sigma_\eta^2 + \sigma_u^2)$, for $t, s = 2, \ldots, T, t \neq s$.

Then, we can write the contribution of the likelihood of observing w_i^{t+1} conditional on w_i^t, E_i^t, w_i^0, on the exogenous regressors $\mathbf{x}_i^t$ and $\boldsymbol{z_i}$, on the unobserved heterogeneity η_i and on all the parameters of the model. Using Equation 2, we define five probabilities: the probability that the individual receives an offer between t and $t+1$ that is superior to his wage in t ($w_i^{t+1} > w_i^t$), the probability that the individual is both working at t and $t+1$ at the same wage rate ($w_i^{t+1} = w_i^t | w_i^t > b$), the probability that the individual is unemployed both in t and in $t+1$ and earns social welfare ($w_i^{t+1} = w_i^t | w_i^t = b$), the

[13] Arulampalam & Stewart (2009) compare three parametric estimation methods to address initial condition issues in non-linear dynamic settings: the Heckman method, the Orme method and the Wooldridge method. They do not find one to be clearly superior to the others.

probability that the individual has lost his job at t but simultaneously receives a job offer for $t+1$ that is inferior to his wage in t but still superior to the social welfare b and that he accepts ($w_i^{t+1} < w_i^t | w_i^{t+1} > b$), and finally the probability that the individual loses his job without finding a new one between t and $t+1$ ($w_i^{t+1} = b | w_i^t > b$). Then, the conditional contribution of individual i to the likelihood at time $t+1$ can be written as:

$$
\begin{aligned}
L_i^{t+1} &= L(w_i^{t+1}|w_i^t, E_i^t, w_i^0, \eta_i, \mathbf{x}_i^t, \boldsymbol{z_i}, \Xi) \qquad (3)\\
&= \left[\frac{\gamma_i^{t+1}}{\sigma_u}\phi\left(\frac{\ln(w_i^{t+1}) - \left(\mathbf{x}_i^t\beta + \lambda\ln(w_i^t) + \tau E_i^t + \theta\ln w_i^0 + \boldsymbol{z_i}\boldsymbol{\rho} + \eta_i\right)}{\sigma_u}\right)\right]^{\mathbb{1}\{w_i^{t+1}>w_i^t\}}\\
&\times \left[(1-\delta_i^{t+1})(1-\gamma_i^{t+1}) + (1-\delta_i^{t+1})\gamma_i^{t+1}\Phi\left(-\frac{\mathbf{x}_i^t\boldsymbol{\beta} + (\lambda-1)\ln(w_i^t) + \tau E_i^t + \theta\ln w_i^0 + \boldsymbol{z_i}\boldsymbol{\rho} + \eta_i}{\sigma_u}\right)\right]^{\mathbb{1}\{w_i^{t+1}=w_i^t|w_i^t>b\}}\\
&\times \left[(1-\gamma_i^{t+1}) + \gamma_i^{t+1}\Phi\left(-\frac{\mathbf{x}_i^t\boldsymbol{\beta} + (\lambda-1)\ln(w_i^t) + \tau E_i^t + \theta\ln w_i^0 + \boldsymbol{z_i}\boldsymbol{\rho} + \eta_i}{\sigma_u}\right)\right]^{\mathbb{1}\{w_i^{t+1}=w_i^t|w_i^t=b\}}\\
&\times \left[\frac{\delta_i^{t+1}\gamma_i^{t+1}}{\sigma_u}\phi\left(\frac{\ln(w_i^{t+1}) - \left(\mathbf{x}_i^t\boldsymbol{\beta} + \lambda\ln(w_i^t) + \tau E_i^t + \theta\ln w_i^0 + \boldsymbol{z_i}\boldsymbol{\rho} + \eta_i\right)}{\sigma_u}\right)\right]^{\mathbb{1}\{w_i^{t+1}<w_i^t|w_i^{t+1}>b\}}\\
&\times \left[\delta_i^{t+1}(1-\gamma_i^{t+1}) + \delta_i^{t+1}\gamma_i^{t+1}\Phi\left(-\frac{\mathbf{x}_i^t\boldsymbol{\beta} + \lambda\ln(w_i^t) - \ln(b) + \tau E_i^t + \theta\ln w_i^0 + \boldsymbol{z_i}\boldsymbol{\rho} + \eta_i}{\sigma_u}\right)\right]^{\mathbb{1}\{w_i^{t+1}=b|w_i^t>b\}}
\end{aligned}
$$

with ϕ and Φ denoting, respectively, the probability and cumulative density functions of the standard normal distribution and Ξ, the set of all parameters of the model.

Then, we integrate out η_i to obtain the conditional contribution of individual i to the likelihood of the model that is the density of $(w_i^1, w_i^2, ..., w_i^T)$ given $(w_i^0, \mathbf{x}_i, \boldsymbol{z_i}, E_i, \Xi)$:

$$L((w_i^1, w_i^2, ..., w_i^T)|w_i^0, \mathbf{x}_i, \boldsymbol{z_i}, E_i, \Xi) = \int\left(\prod_{t=1}^{T} L(w_i^t|w_i^{t-1}, E_i^{t-1}, w_i^0, \eta_i, \mathbf{x}_i^t, \boldsymbol{z_i}, \Xi)\right)\frac{1}{\sigma_\eta}\phi(\eta_i)d\eta$$

where the integral will be computed using the Gaussian-Hermite quadrature.

Then, we sum the log transformation of each contribution over all individuals to obtain the log-likelihood of the model:

$$L = \sum_{i=1}^{N}\ln\left[\int\left(\prod_{t=1}^{T} L(w_i^t|w_i^{t-1}, E_i^{t-1}, w_i^0, \eta_i, \mathbf{x}_i^t, \boldsymbol{z_i}, \Xi)\right)\frac{1}{\sigma_\eta}\phi(\eta_i)d\eta\right]. \qquad (4)$$

One could argue that the random effect corrects for the endogeneity of w_i^{t-1}, but not for the possible endogeneity of the number of employed peers, E_i^{t-1}. As a robustness check, we estimate a joint model for the wage dynamics and the number of employed friends where we permit correlated random effects on two endogenous variables (an individual's number of employed peers and his previous wage).[14] That is, we add to the previous estimation an ordered probit model of the number of employed peers:

$$
\begin{aligned}
\ln(w_i^{t+1}) &= f_w(w_i^t, E_i^t, w_i^0, \mathbf{x}_i^t, \boldsymbol{z_i}, \eta_i, \Xi, u_i^t) \\
E_i^{t+1} &= 0 \text{ if } e_0 w_i^t + e_1 E_i^t + \mathbf{x}_{2i}^t \boldsymbol{e} + \nu_i + u_{2i}^t < a_0 \\
&= 1 \text{ if } a_0 \leq e_0 w_i^t + e_1 E_i^t + \mathbf{x}_{2i}^t \boldsymbol{e} + \nu_i + u_{2i}^t < a_1 \\
&= 2 \text{ if } a_1 \leq e_0 w_i^t + e_1 E_i^t + \mathbf{x}_{2i}^t \boldsymbol{e} + \nu_i + u_{2i}^t < a_2 \\
&= 3 \text{ if } a_2 \leq e_0 w_i^t + e_1 E_i^t + \mathbf{x}_{2i}^t \boldsymbol{e} + \nu_i + u_{2i}^t
\end{aligned}
$$

where the random effects ν_i and η_i are jointly normally distributed with variances σ_ν and σ_η, and correlation $\rho_{\nu\eta}$. The errors (u_i^t, u_{2i}^t) are assumed to be independent and normally distributed, with variances σ_u to be estimated and σ_{u_2} fixed at 1.[15]

We now briefly present the data.

3.1. Data

We use the BHPS, covering the period 2000-2006, to examine the wage dynamics of British men and women. This panel is a nationally representative sample of households whose members are re-interviewed each year.

We focus our analysis on the 2000-2006 period to avoid the changes in wage distribution due to the introduction of the minimum wage in the UK in April 1999 and to limit attrition[16]. The sample is restricted to 18-to 65-year-old who are in the labour force during the whole period (we thus exclude retired individuals, full-time students and individuals in family care). We drop all observations with missing information on usual gross pay per month and the number of hours normally worked per week. We trim the top and bottom 1% tails of wages and working hours, and we compute the hourly wage by dividing the usual gross pay per month by the number of hours normally worked per month. Finally, wages are deflated by the consumer price index and computed in 2008 British pounds. We also observe the year in which individuals begin their current job. We use that information to identify changes in wages between two periods that come from a job offer or termination, and not from measurement errors or salary raises.

For all individuals, we observe their education level, age, marital status and the employment status of their three best friends. The survey also collects information on

[14] Stewart (2007) also estimates a joint bivariate probit model with correlated error terms and random effects.

[15] Qu & Lee (2015) also use a similar strategy for modeling the dependence between the network and the outcome equation.

[16] Over a period of 7 years, 61% of men and 65% of women present in 2000 are still present in 2006. Attrition is not completely orthogonal to characteristics. Individuals who leave the panel are in average younger and so are less employed, have lower wages and are less married.

individuals' health status, which is a dummy variable indicating whether the health of an individual limits the type or amount of work he could perform. We add the regional unemployment rate for each period to the panel.[17] Finally, we also use additional information on the declared friends (age, sex) and on the type of the relationship: whether the friend is a relative and how long has the relationship lasted (more or less than 2 years). Information of friends is only available every two years. We impute missing data on friends using data from the previous year.

We keep observations for individuals who provide information on at least one friend each year; we obtain a balanced panel of 1694 men and 2458 women present all years[18]. Descriptive statistics are presented in Tables 1, 2, 3 and 4. Table 1 shows the evolution of individuals' characteristics overtime. As people age, individuals get married, have more health problem and obtain higher hourly wages. However their employment status, the number of friends and their friends'employment status fluctuate overtime. There is no trend for the unemployment rate. We observe a small increase in the average education level meaning individuals acquire education overtime. In the further analysis, we fix their education level as a constant overtime equal to the highest education level reached over the period[19].

More than 90% of individuals declare having at least three best friends. 55% of men have three best friends employed, whereas it is the case for only 36% of women. Among employed friends, table 4 shows that 85% of women's employed friends are women and 80% of men's employed friends are men. For both sexes, 8% of employed friends are new friends and 65% of women's employed friends are younger and 68% of men's employed friends are younger. These numbers do not vary by education level. However, the share of relatives among employed friends varies by sex and education level. This share is higher for women and for people with low education. It varies from 14% for men with some college education to 27% for women with a low education level (High school diploma or less).

3.2. Results

Consistently with the literature, we find little qualitative impacts of controlling for self-selection in the social network.[20] In order to lighten presentation of the results, we

[17]Twelve regions are reported: North East, North West, Yorkshire and the Humber, East Midlands, West Midlands, East of England, London, South East, South West, Wales, Scotland and Northern Ireland.

[18]In the initial data, women are slightly over-represented (53.5%), when we restrict the sample to individuals who declare at least one friend, women's share increases to 54.5%. The panel attrition is stronger for men so that dropping individuals not present in all years increases the share to 56%. Finally, missing values on wages are more numerous among men so that the women's share in our final sample is 59%

[19]Full time student are excluded and people outside the labour force (e.g. long term disabled) are also excluded.

[20]See Boucher & Fortin (2015) for a discussion.

therefore present the joint model as a robustness check and postpone its analysis to the end of this section.

The estimates of the parameters are presented in Table 5 for women and in Table 6 for men. In each table, we compare three models: one with uncorrelated random effects where initial conditions are assumed to be exogenous, a second with uncorrelated random effects where we include a dependency to the initial wage, and a third with correlated random effects and a dependency to the initial conditions.[21]

We find a job destruction rate (δ) of 16.5% for women with zero employed friends and of 11.0% for men. One additional employed friend decreases the job destruction rate by 1.9 percentage point for women but has no impact for men. We find a rate of offers (γ) of 7.3% for women with no employed friends and of 18.5% for men. One additional employed friend increases the job offer rate by 7.5% point of percentage for women and by 8.5% for men. So that one additional employed friend increases the job offer rate of women without employed friends by 100% and the job offer rate of men by 50%.

The hypothesis $\theta = 0$, exogeneity of the initial condition, is strongly rejected (column 2 of tables 5 and 6). A 10 % increase in the initial wage increases the average wage of one individual's job by 1.1% for women and by 1.6% for men. When we relax the hypothesis of the exogeneity of initial conditions, the effect of the current wage on the distribution of offers decreases significantly (columns 2 and 3). However, we still find a small but positive and significant effect of the current wage on the average distribution of job offers. A 10% increase in the current wage increases the average wage of an individual's job offers by 0.24% for women and by 0.79% for men.

There is a significant negative impact of the number of employed friends in the third model (column 3 of Table 5 and 6) for both men and women. Whereas having an additional employed friend strongly increases the probability to receive a job offer and decreases for women the destruction rate of the job, it has a negative impact on the average wage of an individual's job offers. It decreases it by 2.5% for women and 2.1% for men. We interpret it as an effect of mismatch between the job and the workers who matched through a friend's contact. Cappellari & Tatsiramos (2015) also find such a negative effect on the first wage obtained after unemployment.

Not surprisingly, a high unemployment rate[22] and having minimal education have a negative effect on the distribution of job offers. We find no impact of the single status and no impact of the health status on the short term[23]. However, having bad health

[21] Correlated random effects are computed by including the means of the exogenous variables over the periods and their interactions with the initial wage.

[22] We consider our sample small enough so that one's individual's employment status has no impact on the area's unemployment rate

[23] Other studies working on the BHPS (Stewart, 2007; Arulampalam & Stewart, 2009) find similar

status for several periods has a strong negative impact on the average distribution of job offers. The proportion of the error variance due to the individual-specific effects amounts to 59% (it corresponds to the cross-period correlation for the composite error term $corr(v_i^t, v_i^s) = \sigma_\eta^2/(\sigma_\eta^2 + \sigma_u^2)$).

Heterogeneity

Whereas gender differences in the density and composition of social networks have been studied in the literature (Ioannides & Loury, 2004), it would also be interesting to understand gender differences in the use and the efficiency of social contacts for improving labour-market outcomes.

To better understand our results, we separate the effect of different type of friend (male friends, female friends, friends who are relatives, friends who are younger, new friends). We denote E^s the number of employed friends of a specific type and we consider the two following specifications.

Specification 1 (friend's type effect on wage). We specify the effect of employed friends on the average of wage offer as $\tau E_i^{t-1} + \tau_s E_i^{s,t-1}$, we specify the effect job offer rate as $\gamma_{it} = \gamma + \gamma_{EF} E_i^{t-1}$ and we fix $\delta_{it} = \delta$.

Specification 2 (friend's type effect on offer rate). We specify the effect of employed friends on wage ass τE_i^{t-1}, and we specify the job offer rate as $\gamma_{it} = \gamma + \gamma_{EF} E_i^{t-1} + \gamma_s E_i^{s,t-1}$ and we fix $\delta_{it} = \delta$.

Results of specification 1 are presented on table 7. Results for specification 2 are presented on table 8. Table 7 shows that different types of friends affect differently men and women wage offers. For women, the negative mismatch effect is particularly strong when the additional employed friend is a woman or a new friend, whereas the negative mismatch effect disappear when the additional employed friend is a man. When we look at men, the negative effect disappears for new friends. It is however strongly negative with women friends and relative friends.

Table 8 shows that having a new employed friend has a strong positive effect on the probability to receive a job offer. Having a man employed friend has a higher positive effect than having a woman employed friend, for both genders but particularly for women. Having a relative employed friend has a lower positive effect on the job offer rate. The effect is particularly low for women.

Our results are in line with those of Cappellari & Tatsiramos (2015). Having one additional employed friend increases the job offer rate but decreases the average wage of

results.

the job offer[24]. Non-relative friends seem to be more helpful and women benefit more from their male employed friends than their women employed friends.

Finally, we estimate a joint model for wage dynamics and for the number of friends employed. Results are presented in table 9 and table 10. We find a non-significant effect of the network on the average wage of job offers. Other coefficients are remarkably stable. In the friend employment equation, we find that the respondent's wage increases the probability to have more employed friend the next period. We computed marginal effects. An increase of 10% in a woman's wage (resp man's wage) decreases the probability to have zero employed friend by 0.5 percentage point (0.4 p.p), increases the probability to have one employed friend by 0.4 p.p (0.3 p.p), increases the probability to have two employed friends by 0.1 p.p (0.09 p.p) and has a very small positive effect on the probability to have three employed friends.

We conclude by discussing areas for further research.

4. Conclusion

The empirical literature on the effects of personal networks in the labour market is small but expanding. We contribute to that literature by proposing a non-linear DSAR and estimating the impact of the employment status of an individual's three best friends on the distribution of his job offers. Our structural econometric model is based on our general microeconomic framework, which allows for a large variety of econometric specifications. We discuss some examples below.

We find that the number of employed friends has a positive effect on the distribution of job offers by increasing the probability of receiving job offers while reducing wages. This finding is important, as it introduces dependence between individuals' wages. It also suggests that information about the status of an individual's peers can be as relevant as the information about the peers themselves.

An interesting finding is that the employment status of an individual's peers has a larger effect on the distribution of job offers for women than for men. This suggests that certain groups of individuals (in this case, women) can be more affected by negative shocks on their peers. Determining which groups are more or less exposed to the status of their peers is a promising area of research.

[24] They find a stronger negative effect for low-skilled workers and suggest that low-skilled individuals' networks are less homogeneous which increases the transmission of inadequate job offers. We ran separate analysis on low-skilled individuals ($\leq$ High School) and high-skilled individuals ($>$ High School) and found a non-significant negative effect of one additional employed friend for high-skilled men ($\tau = -0.15$) whereas we found a significant negative effect of $\tau = -0.057$ for low-skilled men. We find a stronger effect for one additional friend who is a relative ($\tau + \tau_s = -0.068$). Results are available on request.

Another potential area for future research (which may be constrained by a lack of available data) is to study the spread of negative aggregate shocks. Our general framework allows for the study of wage dynamics outside the stationary distribution. The fact that wages are positively correlated points to a multiplicative effect of recessions: the total impact is a combination of both the direct impact of a shock, as well as the indirect impact that occurs through the employment status of individuals' peers.

Throughout the paper, we assume that the social network structure is fixed and independent of wage dynamics. This is consistent with our empirical application, since wages are unlikely to be a significant determinant of close friendships. However, some networks (e.g. co-workers) are much more likely to be determined as a function of labour-market outcomes. This raises interesting and challenging questions as to the extent to which individuals are strategic in choosing their friends in time-varying endogenous social networks.

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6. Appendix

6.1. Proofs

PROOF (OF PROPOSITION 1). The proof follows directly from theorem 4.3.9 from Müller & Stoyan (2002).

PROOF (OF PROPOSITION 2). See Müller & Stoyan (2002), theorem 3.10.7 and theorem 4.3.13.

PROOF (OF PROPOSITION 3). The proof is based on the theory of independencies for Markov network models. The reference used here is Koller & Friedman (2009), section 4.3.

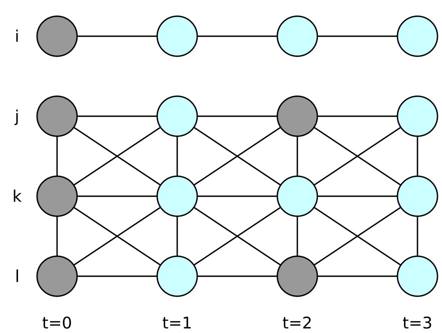
Let us consider the set of all wages, for all individuals, at any point in time from $t = 0$ to $t = T$, conditional on the characteristics $\mathbf{X}^t$, i.e. $\{w_1^t, ..., w_n^T | (\mathbf{X}^{T-1})\}$.

We now define a Markov network structure. Let $\mathcal{H}$ be an undirected graph where a typical node is (i, t) for $i \in N$ and $0 \leq t \leq T$. Consider (i, t) and (j, τ) in $\mathcal{H}$. We set $\tau \geq t$ without a loss of generality. We assume that a link exists between (i, t) and (j, τ) if one of the following conditions holds:

1. $i = j$ and $\tau = t - 1$
2. $j \in N^d(i)$ and $\tau = t - 1$
3. $j \in N^d(i)$ and $\tau = t$

We provide an example in Figure 2.

Figure 2: Markov Network



Given an arbitrary graph $\mathcal{G}$ on (i, t), the Markov blanket of a generic random variable $\mathbf{Z} = \{z_i^t\}$ is defined as:

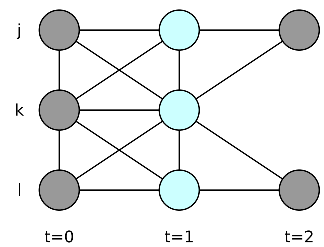
$$I(\mathcal{G}) = \{(z_i^t \perp \mathbf{Z} - \{z_j^\tau\}_{(j,\tau),(i,t)\in\mathcal{G}} - z_i^t) | \{z_j^\tau\}_{(j,\tau),(i,t)\in\mathcal{G}}\}$$

Conditional on the neighbours of (i, t) in $\mathcal{G}$, the realizations on z_i^t are independent of the other variables in $\mathbf{Z}$.

In the context of $\{w_1^t, ..., w_n^T | (\mathbf{X}^{T-1})\}$ and $\mathcal{H}$, Assumption 2 implies that $\mathcal{H}$ summarizes the dependence structure of $z_i^t \equiv w_i^t | (\mathbf{X}^{t-1})$. Since Assumption **??** holds, looking at the Markov blanket is sufficient to describe the dependence structure (see Koller & Friedman (2009), corollary 4.1).

For instance, in the example in Figure 2, we see that w_i^1 and w_l^1 are independent, conditional on $\mathbf{w}^0, (\mathbf{X}^0)$ (i.e. there is no path between $(i, 1)$ and $(j, 1)$). However, there are paths between, say, $(j, 2)$ and $(l, 2)$. More importantly, there is at least one path between $(j, 2)$ and $(l, 2)$ that does not pass through $[(i, 0), (j, 0), (k, 0), (l, 0)]$: the path through $(k, 1)$, as shown in Figure 3.

Figure 3: Separation for $(\mathbf{w}^0, w_j^2, w_l^2)$



Then, $w_j^2|(\mathbf{X}^1)$ and $w_l^2|(\mathbf{X}^1)$ are dependent, conditional on $\mathbf{w}^0$. This follows formally from the definition of the dependence network, which implies that $\mathcal{H}$ is the minimal $I-map$ (see Koller & Friedman (2009), theorem 4.6).

The same argument applies in general: $\mathbf{w}^0$ separates $w_i^t|(\mathbf{X}^{t-1})$ and $w_j^t|(\mathbf{X}^{t-1})$ if and only if the shortest path between i and j in the dependence network is greater than t, i.e. iff $\rho^d(i,j) > t$. QED

Proof (of Proposition 4). The proof is based on Dorea & Pereira (2006). From Theorems 2 and 3, it is sufficient to show that there exists a probability μ_t, an integer $m_t \geq 1$, and constants $\alpha_t < 1/2$ and $\beta_t > 0$ such that for any $A \in \mathcal{B}^n$, $\mu_t(A) > \alpha_t$ implies that

$$\inf_{\mathbf{w}\in W} P^{m_t}(\mathbf{w}^0, (\mathbf{X}^{m_t-1}), A) \geq \beta_t \tag{5}$$

We have:

$$\begin{aligned} P^{m_t}(\mathbf{w}^0, (\mathbf{X}^{m_t-1}), A) &= \int_W P(\mathbf{w}, \mathbf{X}^{m_t-1}, A) P^{m_t-1}(\mathbf{w}^0, (\mathbf{X}^{m_t-2}), d\mathbf{w}) \\ &\geq P(\mathbf{b}, \mathbf{X}^{m_t-1}, A) P^{m_t-1}(\mathbf{w}^0, (\mathbf{X}^{m_t-2}), \mathbf{b}) \\ &\geq P(\mathbf{b}, \mathbf{X}^{m_t-1}, A)\underline{\delta} \end{aligned}$$

where the last inequality follows from Assumption 3. Letting $\mu_t(A) = P(\mathbf{b}, \mathbf{X}^{m_t-1}, A)$ completes the proof. QED

6.2. Tables

Table 1: Summary statistics

	2000	2003	2006
Women			
Age	39.01	42.02	45.01
	(11.66)	(11.66)	(11.66)
Education Level	3.199	3.283	3.343
	(1.602)	(1.658)	(1.700)
Single status	0.275	0.259	0.253
	(0.446)	(0.438)	(0.435)
Health status	0.146	0.156	0.175
	(0.353)	(0.363)	(0.380)
Number of friends	2.890	2.891	2.851
	(0.382)	(0.379)	(0.444)
Employment status	0.680	0.693	0.679
	(0.466)	(0.462)	(0.467)
Number of employed friends	1.996	2.004	1.983
	(0.948)	(0.940)	(0.968)
Unemployment rate	5.729	4.962	5.313
	(1.346)	(0.914)	(0.798)
Hourly wage	6.359	7.150	7.505
	(5.055)	(5.658)	(6.055)
Men			
Age	38.41	41.42	44.41
	(11.73)	(11.72)	(11.73)
Education Level	3.512	3.586	3.626
	(1.675)	(1.712)	(1.735)
Single status	0.297	0.264	0.238
	(0.457)	(0.441)	(0.426)
Health status	0.131	0.144	0.156
	(0.338)	(0.351)	(0.363)
Number of friends	2.882	2.879	2.836
	(0.410)	(0.405)	(0.469)
Employment status	0.826	0.832	0.820
	(0.379)	(0.374)	(0.384)
Number of employed friends	2.312	2.321	2.274
	(0.901)	(0.876)	(0.912)
Unemployment rate	5.679	4.942	5.304
	(1.348)	(0.914)	(0.787)
Hourly wage	9.867	10.80	11.39
	(6.469)	(7.556)	(7.785)

Sample averages by year. Standard Deviations are in parenthesis.

Table 2: Summary statistics for the number of declared friends. Women

	≤ High School	**> High School**	**All**
Number of Friends			
One	0.04	0.01	0.03
Two	0.08	0.05	0.07
Three	0.88	0.94	0.90
Number of Employed Friends			
Zero	0.11	0.05	0.08
One	0.22	0.15	0.20
Two	0.35	0.36	0.36
Three	0.32	0.44	0.36

Table 3: Summary statistics for the number of declared friends. Men

	≤ High School	**> High School**	**All**
Number of Friends			
One	0.04	0.02	0.03
Two	0.07	0.06	0.06
Three	0.89	0.92	0.91
Number of Employed Friends			
Zero	0.06	0.04	0.05
One	0.14	0.12	0.13
Two	0.28	0.26	0.27
Three	0.52	0.58	0.55

Table 4: Share of friends'type among employed friends by education level and sex

	Women		**Men**	
	≤ High School	**> High School**	**≤ High School**	**> High School**
Relative friends	0.268	0.226	0.186	0.143
Same-sex friends	0.855	0.841	0.793	0.806
New friends	0.081	0.084	0.077	0.077
Younger friends	0.646	0.620	0.680	0.671

Table 5: Estimates on Women. 2000-2006

	RE exogeneity of IC	**RE IC**	**CRE IC**
Age	0.025	0.013	0.019
	(0.002)	*(0.002)*	*(0.004)*
Age^2	-0.000	-0.000	-0.000
	(0.000)	*(0.000)*	*(0.000)*
Education level	0.117	0.116	0.115
	(0.010)	*(0.008)*	*(0.011)*
Single status	0.037	0.052	0.045
	(0.024)	*(0.024)*	*(0.034)*
Health status	-0.027	-0.038	-0.006
	(0.037)	*(0.038)*	*(0.042)*
Unemployment rate	-0.027	-0.039	-0.055
	(0.010)	*(0.011)*	*(0.015)*
Hourly wage (t-1)	0.061	0.026	0.024
	(0.011)	*(0.011)*	*(0.012)*
Number of Employed friends (t-1)	-0.025	-0.022	-0.025
	(0.012)	*(0.012)*	*(0.013)*
δ	0.165	0.165	0.165
	(0.010)	*(0.010)*	*(0.010)*
δ_{EF}	-0.019	-0.019	-0.019
	(0.004)	*(0.004)*	*(0.004)*
γ	0.073	0.073	0.073
	(0.006)	*(0.006)*	*(0.006)*
γ_{EF}	0.075	0.075	0.075
	(0.004)	*(0.004)*	*(0.004)*
σ_u	0.294	0.289	0.288
	(0.006)	*(0.006)*	*(0.007)*
Constant	1.263	1.486	1.131
	(0.088)	*(0.088)*	*(0.178)*
σ_η	0.355	0.353	0.354
	(0.010)	*(0.010)*	*(0.016)*
Initial hourly wage		0.109	0.192
		(0.014)	*(0.066)*
$\overline{\text{Single status}}$			0.094
			(0.060)
$\overline{\text{Health status}}$			-0.209
			(0.095)
$\overline{\text{Unemployment rate}}$			0.062
			(0.034)
Initial hourly wage *$\overline{\text{Single status}}$			-0.045
			(0.032)
Initial hourly wage *$\overline{\text{Health status}}$			0.080
			(0.059)
Initial hourly wage *$\overline{\text{Unemployment rate}}$			-0.015
			(0.012)
Loglikelihood	-9477	-9446	-9440
$N = 2458$			

*** p<0.01, ** p<0.05, * p<0.1

Table 6: Estimates on Men. 2000-2006

	RE exogeneity of IC	**RE IC**	**CRE IC**
Age	0.056	-0.015	-0.012
	(0.001)	*(0.001)*	*(0.001)*
Age^2	-0.001	0.000	-0.000
	(0.000)	*(0.000)*	*(0.000)*
Education Level	0.129	0.119	0.107
	(0.008)	*(0.008)*	*(0.007)*
Single status	-0.049	-0.032	-0.032
	(0.030)	*(0.028)*	*(0.039)*
Health status	0.022	0.036	0.048
	(0.036)	*(0.035)*	*(0.044)*
Unemployment rate	-0.034	-0.025	-0.045
	(0.010)	*(0.010)*	*(0.015)*
Hourly wage (t-1)	0.101	0.086	0.079
	(0.013)	*(0.013)*	*(0.013)*
Number of Employed friends (t-1)	-0.012	-0.018	-0.021
	(0.014)	*(0.014)*	*(0.013)*
δ	0.111	0.111	0.110
	(0.011)	*(0.011)*	*(0.011)*
δ_{EF}	-0.004	-0.004	-0.004
	(0.004)	*(0.004)*	*(0.004)*
γ	0.133	0.132	0.185
	(0.013)	*(0.017)*	*(0.013)*
γ_{EF}	0.081	0.081	0.085
	(0.006)	*(0.008)*	*(0.006)*
σ_u	0.282	0.280	0.277
	(0.006)	*(0.006)*	*(0.006)*
Constant	0.758	1.799	2.273
	(0.096)	*(0.081)*	*(0.190)*
σ_η	0.358	0.350	0.423
	(0.009)	*(0.008)*	*(0.007)*
Initial hourly wage		0.159	-0.024
		(0.017)	*(0.072)*
$\overline{\text{Single status}}$			0.277
			(0.078)
$\overline{\text{Health status}}$			0.270
			(0.100)
$\overline{\text{Unemployment rate}}$			0.100
			(0.034)
Initial hourly wage *$\overline{\text{Single status}}$			0.051
			(0.030)
Initial hourly wage *$\overline{\text{Health status}}$			-0.171
			(0.042)
Initial hourly wage *$\overline{\text{Unemployment rate}}$			-0.019
			(0.013)
Loglikelihood	-6583	-6553	-6480
N= 1694			

*** p<0.01, ** p<0.05, * p<0.1

Table 7: Effect of employed friends on wages

	$\tau + \tau_s$	γ_{EF}	γ	loglikelihood
Women				
Men Friends	0.007	0.075	0.073	-9448
	(0.020)	*(0.004)*	*(0.006)*	
Women Friends	-0.029	0.075	0.073	-9448
	(0.013)	*(0.004)*	*(0.006)*	
Relative Friends	-0.022	0.075	0.073	-9450
	(0.019)	*(0.004)*	*(0.006)*	
New Friends	-0.031	0.075	0.073	-9450
	(0.017)	*(0.004)*	*(0.006)*	
Younger Friends	-0.025	0.075	0.073	-9450
	(0.014)	*(0.004)*	*(0.006)*	
Men				
Men Friends	-0.017	0.085	0.185	-6480
	(0.014)	*(0.006)*	*(0.013)*	
Women Friends	-0.033	0.085	0.185	-6480
	(0.019)	*(0.006)*	*(0.013)*	
Relative Friends	-0.033	0.085	0.185	-6480
	(0.017)	*(0.006)*	*(0.013)*	
New Friends	-0.007	0.082	0.185	-6480
	(0.022)	*(0.006)*	*(0.013)*	
Younger Friends	-0.014	0.085	0.185	-6480
	(0.014)	*(0.006)*	*(0.013)*	

Table 8: Effect of employed friends on job offer rate

	τ	$\gamma_{EF} + \gamma_s$	γ	loglikelihood
Women				
Men Friends	-0.024	0.100	0.073	-9444
	(0.013)	*(0.009)*	*(0.006)*	
Women Friends	-0.034	0.072	0.082	-9464
	(0.014)	*(0.004)*	*(0.008)*	
Relative Friends	-0.026	0.030	0.076	-9409
	(0.013)	*(0.006)*	*(0.007)*	
New Friends	-0.036	0.164	0.083	-9435
	(0.014)	*(0.012)*	*(0.009)*	
Younger Friends	-0.025	0.064	0.073	-9440
	(0.012)	*(0.004)*	*(0.006)*	
Men				
Men Friends	-0.021	0.087	0.185	-6480
	(0.013)	*(0.008)*	*(0.017)*	
Women Friends	-0.017	0.070	0.186	-6480
	(0.013)	*(0.014)*	*(0.017)*	
Relative Friends	-0.018	0.066	0.185	-6479
	(0.013)	*(0.014)*	*(0.017)*	
New Friends	-0.017	0.143	0.185	-6473
	(0.013)	*(0.017)*	*(0.017)*	
Younger Friends	-0.018	0.075	0.186	-6478
	(0.013)	*(0.009)*	*(0.017)*	

Table 9: Joint model of the number of employed friends and wage dynamics. Women

Wage Equation		Friend employment equation	
Age	0.020	Unemployment rate	-0.009
	(0.002)		*(0.011)*
Age^2	-0.0003	Hourly Wage (t-1)	0.209
	(0.0000)		*(0.012)*
Education Level	0.116	Number of employed	1.259
	(0.007)	friends (t-1)	*(0.015)*
Single Status	0.044	a_0	0.271
	(0.034)		*(0.064)*
Health Status	0.001	a_1	1.803
	(0.041)		*(0.065)*
Unemployment rate	-0.055	a_2	3.482
	(0.015)		*(0.069)*
Hourly Wage (t-1)	0.021	σ_ν	0.353
	(0.012)		*(0.004)*
Number of employed friends (t-1)	-0.012		
	(0.012)		
δ	0.165		
	(0.010)		
δ_{EF}	-0.019		
	(0.004)		
γ	0.073		
	(0.007)		
γ_{EF}	0.074		
	(0.004)		
σ_u	0.288		
	(0.005)		
Constant	1.116		
	(0.128)		
$\overline{\text{Single status}}$	0.084		
	(0.057)		
$\overline{\text{Health status}}$	-0.220		
	(0.084)		
$\overline{\text{Unemployment rate}}$	0.061		
	(0.026)		
Initial hourly wage	0.196		
	(0.067)		
Initial hourly wage *$\overline{\text{Single status}}$	-0.041		
	(0.027)		
Initial hourly wage *$\overline{\text{Health status}}$	0.091		
	(0.054)		
Initial hourly wage *$\overline{\text{Unemployment rate}}$	-0.016		
	(0.012)		
σ_η	0.353		
	(0.008)		
$\rho_{\nu\eta}$	-0.140		
	(0.067)		
Log likelihood	-21758		
N	2458		

*** p<0.01, ** p<0.05, * p<0.1

Table 10: Joint model of the number of employed friends and wage dynamics. Men

Wage Equation		Friend employment equation	
Age	0.0000	Unemployment rate	-0.025
	(0.004)		*(0.014)*
Age^2	-0.0002	Hourly Wage (t-1)	0.165
	(0.0001)		*(0.015)*
Education Level	0.105	Number of employed	1.234
	(0.008)	friends (t-1)	*(0.019)*
Single status	-0.033	a_0	0.108
	(0.039)		*(0.088)*
Health Status	0.049	a_1	1.575
	(0.043)		*(0.088)*
Unemployment rate	-0.048	a_2	3.000
	(0.014)		*(0.092)*
Hourly Wage (t-1)	0.077	σ_ν	0.423
	(0.006)		*(0.004)*
Number of employed friends (t-1)	-0.018		
	(0.013)		
δ	0.111		
	(0.011)		
δ_{EF}	-0.004		
	(0.004)		
γ	0.184		
	(0.017)		
γ_{EF}	0.084		
	(0.008)		
σ_u	0.281		
	(0.005)		
Constant	2.062		
	(0.153)		
$\overline{\text{Single status}}$	0.311		
	(0.041)		
$\overline{\text{Health status}}$	-1.353		
	(0.016)		
$\overline{\text{Unemployment rate}}$	-0.127		
	(0.030)		
Initial hourly wage	-0.033		
	(0.060)		
Initial hourly wage *$\overline{\text{Single status}}$	-0.105		
	(0.037)		
Initial hourly wage *$\overline{\text{Health status}}$	0.554		
	(0.005)		
Initial hourly wage *$\overline{\text{Unemployment rate}}$	0.068		
	(0.008)		
σ_η	0.356		
	(0.010)		
$\rho_{\nu\eta}$	-0.021		
	(0.047)		
Log likelihood	-13859		
N	1694		

*** p<0.01, ** p<0.05, * p<0.1