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Adversarial Persuasion with Cross-Examination

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Abstract

Two parties with opposed interests invest in acquiring evidence which they may only partially disclose. The decision maker then adjudicates. This set-up is compared with one permitting cross-examination of the other party s report. Now the decision maker can better assess whether a report was deceitful through withholding of evidence. Nevertheless, decision-making need not be improved. The parties invest less in gathering evidence because they are less able to successfully manipulate information and because cross-examination is a substitute in potentially countering the other party. From the decision maker's standpoint, there is too much cross-examination at the expense of too little direct evidence.

JEL Classification: disclosure, persuasion, evidence, adversarial, cross-examination, judicial procedures.
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1 Introduction

We study a persuasion game where two parties with opposed interests search for evidence in order to influence an uninformed decision maker. Evidence potentially consists of many pieces of hard information from which a party strategically selects what to disclose. The decision maker then adjudicates. We compare this set-up with one allowing cross-examination, by which we mean raising issues about the other party's report. Now, in the investigation phase, the parties can invest both in acquiring direct evidence and in acquiring means of cross-examination, e.g., identifying key issues that can potentially be raised. In the communication phase, once the parties have submitted evidence, cross-examination elicits information as to whether the other party's report was misleading through withholding of evidence. The decision maker can therefore better assess the significance of reports.

We show that accuracy in adjudication need not be improved. Conditional on the parties' investment in gathering evidence, cross-examination improves communication. However, the parties now invest less in gathering evidence. Given the threat of cross-examination, they anticipate that they will be less able to successfully manipulate information. Moreover, in the investigation phase, acquiring evidence and acquiring means of cross-examination are substitutes in potentially countering the other party. As a result, the quality of inferences from a possibly misleading report may deteriorate. The reason is that inferences from a report also depend on how likely it is that the other party acquired evidence that he did not disclose. From the decision maker's standpoint, there is too much investment in cross-examination at the expense of too little gathering of directly relevant evidence.

Strictly speaking, cross-examination refers to a celebrated feature of the common law trial. It is the interrogation of a witness called by the adverse party after the witness has been subject to direct examination by that party. This is one of the main differences between the common law adversarial procedure and its counterpart in the civilist tradition. In the latter, as concerns civil disputes, trials are adversarial to the extent that it is the parties' responsibility to provide evidence, but there is no or little cross-examination. It is often said that cross-examination is a powerful weapon to ferret out the truth. In the words of Wigmore (1940, § 1387), "it is the greatest legal engine ever invented for the discovery of truth".

We interpret cross-examination in the general sense of actions that seek to lessen the weight of another party's report not by providing directly relevant countervailing evidence but by questioning the 'believability' of the report or its interpretation. In practice, a cross-examiner may question a testimony's reliability, e.g., the witness exhibits confusion and unwillingly misstates facts. A cross-examiner may also question the credibility of a testimony, e.g., the witness contradicts himself and presumably lies. Our analysis, however, follows the economic literature on voluntary disclosure games in assuming that evidence is hard information that can be concealed but cannot be falsified or fabricated (Grossman 1981, Milgrom 1981, Milgrom and Roberts 1986). In this framework, the intrinsic 'statistical' reliability of signals is common knowledge and outright lies are not possible. A report may nevertheless be deceitful because of the suppression of relevant pieces of information.

As is well known from the disclosure game literature, when an interested party's information status is common knowledge, full revelation of private information is induced by the decision maker's skeptical posture of 'assuming the worst'; see Mathis (2008) for a more precise statement and a synthesis of the literature. Full unraveling is also well known to fail, however, when there is a possibility that the interested party possesses no hard information (Dye 1985, Shavell 1989). Competition in persuasion between parties with opposed interests is then generally beneficial to the decision maker, as in Shin (1994a, 1998), Dewatripont and Tirole (1999) or more recently Bhattacharya and Mukherjee (2013).

Our basic set-up is in the spirit of Shin's (1998) model of the adversarial procedure, but with the following features. As in Kim's (2013) extension of that model, we consider situations where the parties' information is endogenous. As in Kartik et al. (2017), who also consider information acquisition, we assume that the parties do not necessarily acquire the same information. This allows for equilibria where, in the investigation phase, both parties invest in acquiring evidence and where, in the communication phase, a party may rebut the other party. In addition, as in Demougin and Fluet (2008), the information acquired by a party may consist of several pieces. When a party submits evidence, the decision maker may remain uncertain whether the submission disclosed the whole truth. This allows for actions, which we refer to as cross-examination, that influence the interpretation of reports by eliciting information about the possible suppression of evidence.

Additional literature connections are reviewed in a discussion section. In particular, our results relate to the effect of actions that reduce the possibility of successfully manipulating information. From the literature on voluntary versus mandatory disclosure, it is well known that reducing the scope of manipulation may be detrimental to the quality of decision making when information is costly and the uninformed party is a sophisticated Bayesian decision maker. There is a trade-off between the agent's incentives to acquire information and the quality of communication conditional on the information acquired. Our results, however, are more intricate because, owing to the adversarial context, the weight given to a possibly misleading report depends on how likely both parties are informed. As a result, communication may itself become less informative.

The paper develops as follows. Section 2 presents the basic set-up. Section 3 analyzes the procedure without the right of cross-examination. Section 4 allows cross-examination and derives the main results under a simplifying assumption ruling out mixed strategies at equilibrium. Section 5 provides a discussion and shows that our results remain essentially unchanged when our simplifying assumption is dropped and equilibria may involve mixed strategies. Section 6 concludes. All proofs are in the Appendix.

2 Model

A decision maker, hereafter the arbitrator, must adjudicate a dispute between two parties P and D, hereafter the plaintiff and the defendant respectively, with diametrically opposed interests. The issue is the value of $\omega \in \{\omega_0, \omega_1\}$ referred to as the underlying fact of the dispute or true state. The state ω_0 favors the defendant, the state ω_1 favors the plaintiff. The arbitrator must make a binary decision $d \in \{0,1\}$ where 0 means that she finds for the defendant and 1 that she finds for the plaintiff. The arbitrator is a disinterested truth-seeker who wants her decision to match the underlying fact. Her payoff is $u(d,\omega) = 1$ if $d = i$ and $\omega = \omega_i$, where $i \in \{0,1\}$, and $u(d,\omega) = 0$ otherwise. The parties' payoffs are state independent. Each party wants the arbitrator to rule in his favor. The plaintiff's payoff from the arbitrator's decision is d, the defendant's payoff is $-d$.

The underlying fact is unknown to the parties and the arbitrator. The prior probability that $\omega = \omega_1$ is p. Without loss of generality we assume $p \leq \frac{1}{2}$, that is, the arbitrator would rule against the plaintiff if she were to rely only on priors.[1] However, evidence is potentially available which consists of the signals $x \in \{x_0, x_1\}$ and $y \in \{y_0, y_1\}$. The joint density with the true state is denoted by $P(\omega, x, y)$. The signals are independent conditionally on the state, $P(x, y \mid \omega) = P(x \mid \omega)P(y \mid \omega)$. We assume

$$P(\ x_0 \mid \omega_0\ ) = P(\ x_1 \mid \omega_1\ ) = q,$$

$$P(\ y_0 \mid \omega_0\ ) = P(\ y_1 \mid \omega_1\ ) = h.$$

We also assume that, by itself, the realization x_1 always yields a posterior favoring the plaintiff despite the prior against him. Moreover, the signal y is more informative. For any realization of x, the joint signal (x, y_1) also yields a posterior favoring the plaintiff. The foregoing amounts to the following restrictions on the precision of the signals.

Assumption 1: $q > 1 - p \geq \frac{1}{2}$ and $h(1-q)p > q(1-h)(1-p)$.

The assumption implies that h is sufficiently larger than q. Applying Bayes' rule, posterior probabilities are

$$P(\omega_1 \mid z) = \frac{pP(z \mid \omega_1)}{pP(z \mid \omega_1) + (1-p)P(z \mid \omega_0)}$$

where $z = x$ or $z = (x, y)$. Table 1 summarizes the implications of Assumption 1.

[1] When $p = \frac{1}{2}$, the arbitrator is indifferent between finding for one party or the other. We assume the plaintiff bears the 'burden of persuasion'. The arbitrator rules in his favor only if $\omega = 1$ is more likely than not.

Table 1
Posterior probabilities as a function of signal realizations

	$P(\omega_1 \mid x)$	$P(\omega_1 \mid x, y_0)$	$P(\omega_1 \mid x, y_1)$
x_0	$< \frac{1}{2}$	$< \frac{1}{2}$	$> \frac{1}{2}$
x_1	$> \frac{1}{2}$	$< \frac{1}{2}$	$> \frac{1}{2}$

The parties can invest in gathering evidence. For simplicity, we assume that they face the same costs. Party $i \in \{P, D]$ obtains some evidence with probability e_i at a cost $C(e_i)$. The function is increasing and strictly convex with convex marginal cost (i.e., $C''' \geq 0$) and with $C(0) = C'(0) = 0$ and $C'(1) \geq 1$. The 'Inada condition' ensures that, at equilibrium and given the stakes, a party will never obtain evidence for sure. With probability e_i, the evidence acquired is x. With probability θe_i, where $\theta \in (0, 1)$, the evidence acquired is (x, y). Conditional on the observation of x, whether y is observed consitutes independent events across the parties. Figure 1 depicts the possible outcome of a party's investement at the investigation phase. Thus, a party may be more or less well informed. A party's investment in gathering evidence is private information. Whether he acquired evidence or what amount of evidence he acquired are also private information. Taking into account the investment in gathering evidence, the parties' net payoffs are

$$\begin{aligned} \pi_P &= d - C(e_P), \\ \pi_D &= -d - C(e_D). \end{aligned}$$

The investigation phase is followed by a communication phase in which the parties may report to the arbitrator. Evidence is hard information that cannot be falsified but may be withheld. The parties' reports are denoted by m_i. If a party was unsuccessful in obtaining evidence, his submission is by force the empty report $m_i = \emptyset$. If the party only acquired the evidence x, his report belongs to the set $\{\emptyset, (x, \emptyset)\}$ where $\emptyset$ means that he submits nothing and $(x, \emptyset)$ that he reports only x. If the party acquired the whole potential evidence (x, y), his report belongs to the set $\{\emptyset, (x, \emptyset), (x, y)\}$.[2] Thus, when a party reports $\emptyset$, the arbitrator does not know whether the party was truly unsuccessful or whether he observed $(x, \emptyset)$ or (x, y). When a party reports $(x, \emptyset)$, the arbitrator does not know whether the party also observed y.

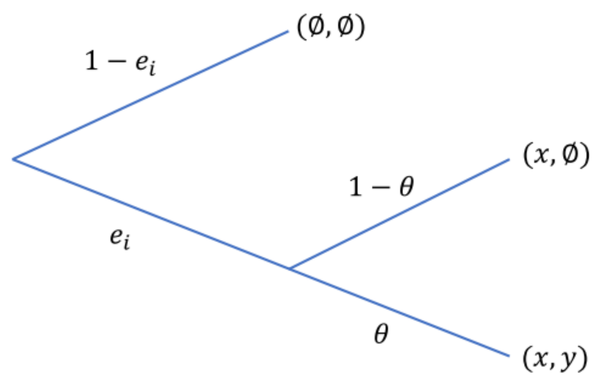

Figure 1. Information Acquisition

The time line is as follows. First, Nature chooses the true state which remains unobservable. Next, the parties simultaneously choose their investment efforts e_P and e_D respectively, which remains private information. At the third stage, they access evidence or not and whether they do also remains private information. Next, the parties simultaneously choose their reports m_P and m_D. At the last stage, the adjudicator observes the reports, updates her beliefs, and adjudicates. We write $\beta(m_P, m_D)$ for the arbitrator's beliefs that the true state is $\omega = \omega_1$ following the parties' reports. Similarly, her adjudication strategy is $d(m_P, m_D) \in \{0, 1\}$. Given the arbitrator's payoff function, the sequentially rational decision is $d(m_P, m_D) = 1$ if and only if $\beta(m_P, m_D) > \frac{1}{2}$.

[2] To simplify, because a party who observes y also observed x, we assume that a party who submits y also simultaneously submits x, i.e., y comes with x 'attached' to it. This reduces the set of possible actions but has no bearing on our results because y is more informative than x.

The preceding time line describes the procedure without the right of cross-examination. We defer to Section 4 the description of the procedure where cross-examination is allowed.

3 Procedure without Cross-Examination

We start with a series of observations on the interpretation of the parties' submissions and how this relates to the equilibria of the persuasion game.

Burden of proof. There are typically multiple equilibria. By assumption priors disfavor the plaintiff or at best are in equipoise. Nevertheless, when p is not too small, there is an equilibrium where the plaintiff prevails in the absence of evidence, i.e., the arbitrator's belief is $\beta(\emptyset, \emptyset) > \frac{1}{2}$. There is also always another equilibrium where the plaintiff loses in the same circumstances, i.e., $\beta(\emptyset, \emptyset) < \frac{1}{2}$. The losing party when no evidence is submitted will be said to bear the burden of proof. This will also be the party with the greatest incentives to acquire evidence, as will become clear. Multiple equilibria arise because acquiring evidence is costly and because the party with the burden is more likely to be better informed. When no evidence is disclosed, the better informed party is therefore the one most likely to have engaged in strategic non-disclosure, which justifies finding against him.

We select the equilibrium where the ex ante disfavored party, the plaintiff in our set-up, bears the burden of proof. The rationale, given the symmetry of investigation costs, is that the risk of adjudication error is then minimized. To illustrate, judicial procedures usually impose the burden of proof on the plaintiff. An interpretation is that the law has settled on the burden assignment that minimizes judicial error for the category of cases considered here. This in turn will be supported by the court's equilibrium belief should no evidence be submitted.[3] Henceforth, we therefore restrict discussion to

[3] Alternatively, the arbitrator understands the incentives created by the burden of proof and purposefully assigns it so as to minimize the risk of error. At a preliminary stage, she announces that she will adjudicate against the plaintiff if no evidence is submitted. Although this is cheap talk, the announcement is credible in the sense that it is both self-committing and self-signaling, hence it should be believed (Farrell and Rabin 1996). The formal argument is the same as in Demougin and Fluet (2008). See also Bull and

equilibria with $\beta(\emptyset, \emptyset) < \frac{1}{2}$, which always exist. If priors favored the plaintiff, all our results would hold *mutatis mutandis* with the reversed burden of proof.[4]

Selective evidence. Because the parties can select what piece of evidence they disclose, the arbitrator's beliefs may depend not only on the submitted piece but also on who disclosed it. Suppose the evidence reduces to x_0 which by itself favors the defendant. If it has been submitted by the defendant while the plaintiff remained silent, i.e., $m_P = \emptyset$ and $m_D = (x_0, \emptyset)$, the arbitrator learns that with probability θ the defendant observed the complete evidence. When the probability is large, the arbitrator may infer that the defendant most likely also observed y but did not report it because it was unfavorable. Hence her belief is $\beta(\emptyset, (x_0, \emptyset)) > \frac{1}{2}$ which goes against the defendant.[5] By contrast, suppose the plaintiff submits the same evidence while the defendant is silent, i.e., $m_P = (x_0, \emptyset)$ and $m_D = \emptyset$. Maybe the plaintiff trembled from what would have been a reasonable strategy. The evidence may then be taken at face value, which favors the defendant. Indeed, should the arbitrator adopt a skeptical stance vis-à-vis the plaintiff, her belief would still be $\beta((x_0, \emptyset), \emptyset) < \frac{1}{2}$.

Even with the burden of proof on the plaintiff, there may still be multiple equilibria. This arises in particular when the plaintiff submits $(x_1, \emptyset)$ and the defendant is silent. The case differs from the above because the arbitrator's response is now pivotal. The plaintiff bears the burden of proof, hence he strictly gains if the arbitrator then rules in his favor. However, if θ is sufficiently large, it is reasonable to infer that the plaintiff most likely observed y but suppressed it. The argument is the same as above. The belief is then $\beta((x_1, \emptyset), \emptyset,) < \frac{1}{2}$. When such a belief is part of the equilibrium, the defendant has no incentive to invest in the acquisition of evidence. Either he wins by default or the plaintiff will anyway submit overpowering evidence. We

Watson (2017) on the court's announcement of its 'interpretive rules'.

[4] When $p > \frac{1}{2}$, Assumption 1 needs to be reformulated to ensure that x_0 is sufficiently strong evidence to counteract the priors against the defendant. Results are otherwise completely symmetrical.

[5] Borrowing from Bull and Watson's (2017) terminology, hard evidence may provide information through its 'face-value signal' and as a signal of the party's private information.

call this a *Passive Defendant* equilibrium, henceforth a *pd*-equilibrium. The communication phase then reduces to a one-sender persuasion game because the arbitrator understands that the defendant cannot provide countervailing evidence.

However, for the same θ provided it is not too large, there is also another equilibrium where the arbitrator's belief is $\beta((x_1, \emptyset), \emptyset,) > \frac{1}{2}$. Now the defendant has an incentive to invest in acquiring evidence. His motivation is that this may possibly counteract the submission of $(x_1, \emptyset)$ by the plaintiff. We call this an *Active Defendant* equilibrium, henceforth an *ad*-equilibrium. Both parties now invest in gathering evidence, hence the communication phase is a two-sender persuasion game. The belief differs from the preceding case because, while the arbitrator understands that the plaintiff may have withheld evidence, she also takes into account the possibility that the defendant could have presented countervailing evidence. As a result, she adopts a less skeptical stance vis-à-vis the plaintiff.

Disclosure strategies. Equilibria may differ in inessential ways with respect to the parties' disclosure strategies. To simplify, we consider a unique profile of strategies consistent with both a *pd* and an *ad*-equilibrium. These strategies are also part of an equilibrium when the right of cross-examination is introduced. In this profile, the plaintiff is the proactive party because he is the one bearing the burden of proof. He always submits 'a priori' favorable evidence and always suppresses 'a priori' unfavorable evidence, where 'a priori' refers to the raw conditional probabilities without sophisticated inferences. The defendant has a minimum disclosure strategy. He only submits (x_1, y_0) if he can.[6] Either it makes no difference for the arbitrator's decision because she would have ruled against the plaintiff anyway or it usefully counteracts the submission of $(x_1, \emptyset)$ by the plaintiff.

Always disclosing 'a priori' favorable and suppressing 'a priori' unfavorable evidence is referred to as a 'sanitization strategy' in Shin (1994a). This is the approach used for both parties in Shin (1994b, 1998) and in Kartik et al. (2017). In their setting, each party can only access a single piece of evidence which is either disclosed whole or not at all. In our case the evidence

[6] Recall that, by convention, y can be reported only with x attached.

comes in two pieces. Selective disclosure reveals that the party has had access to some evidence and therefore subjects the party to the possibility of adverse inferences as described above or to the risk of cross-examination when the procedure allows it. The defendant's strategy of submitting only overpowering evidence avoids these risks.[7]

The plaintiff's disclosure strategy is

$$\begin{cases} (\emptyset,\emptyset),(x_0,\emptyset),(x_0,y_0) & \longmapsto & \emptyset \\ (x_1,\emptyset),(x_1,y_0) & \longmapsto & (x_1,\emptyset) \\ (x_0,y_1) & \longmapsto & (x_0,y_1) \\ (x_1,y_1) & \longmapsto & (x_1,y_1) \end{cases} \tag{1}$$

The defendant's strategy is

$$\begin{cases} (\emptyset,\emptyset),(x_0,\emptyset),(x_1,\emptyset),(x_0,y_0),(x_0,y_1),(x_1,y_1) & \longmapsto & \emptyset \\ (x_1,y_0) & \longmapsto & (x_1,y_0) \end{cases} \tag{2}$$

In a *pd*-equilibrium, the defendant does not investigate and therefore will not be in a position to report along the equilibrium path. Nevertheless, the above strategy describes what he would do should he investigate and obtain evidence.

Many pairs of testimonies never arise on the equilibrium path. For such pairs, beliefs cannot be derived from Bayes' rule and are obtained as the limit of completely mixed strategies where the parties play out-of-equilibrium moves with some small probability.[8]

Lemma 1 *Under the disclosure strategies (1) and (2), the arbitrator's beliefs disfavor the plaintiff when the evidence reduces to* $(x_0,\emptyset)$. *When it reduces to* $(x_1,\emptyset)$ *submitted by the defendant, beliefs favor the plaintiff. When the complete evidence* (x,y) *is disclosed, the beliefs are the naïve posteriors.*

To connect with a previous observation, observe that the lemma implies $\beta(\emptyset,(x_0,\emptyset)) < \frac{1}{2}$ irrespective of θ. The defendant's report is seen as a random

[7] Our disclosure strategies are also part of an equilibrium when the parties play sequentially in the communication phase, with the plaintiff moving first and the defendant reacting (if at all) to the evidence submitted by the plaintiff.

[8] Our Perfect Bayesian equilibria are therefore also sequential equilibria (cf. Fudenberg and Tirole 1991).

deviation from his equilibrium disclosure strategy. If this action were part of an equilibrium strategy while (x_0, y_1) is not, θ would matter.

Investigation stage. From Lemma 1, the arbitrator rules in the plaintiff's favor only if the evidence is (x_0, y_1) or (x_1, y_1) and possibly also if it reduces to $(x_1, \emptyset)$. So far, the arbitrator's strategy $d(m_P, m_D)$ is therefore completely defined except for $m_P = (x_1, \emptyset)$ and $m_D = \emptyset$.

At the investigation stage, the parties's expected payoffs are

$$\begin{aligned} \pi_P &= S(e_D)e_P - C(e_P), &(3)\\ \pi_D &= -S(e_D)e_P - C(e_D), &(4) \end{aligned}$$

where

$$\begin{aligned} S(e_D) \equiv\ & P(x_0, y_1)\theta + P(x_1, y_1)\left[\theta + (1-\theta)d((x_1, \emptyset), \emptyset)\right] \\ & + P(x_1, y_0)(1 - e_D\theta)d((x_1, \emptyset), \emptyset) \qquad (5) \end{aligned}$$

is the probability that the plaintiff succeeds conditional on having access to some evidence and where $P(x, y)$ is the marginal distribution of the potential evidence. Because each party's payoff is concave in his own investigation effort, we can discard mixed strategies over effort levels. Accordingly, in (3), e_D can be interpreted as the plaintiff's deterministic conjecture of the defendant's investment in gathering evidence. Similarly, in (4), e_P is the defendant's conjecture of the plaintiff's investment.[9]

The expression in (5) follows from the disclosure strategies. Given that he has access to some evidence, the plaintiff prevails in the following circumstances: (i) the evidence is (x_0, y_1) and the plaintiff obtained the complete evidence, which occurs with probability θ; (ii) the potential evidence is (x_1, y_1) and the plaintiff has access to the complete evidence or he only has access to partial evidence but the arbitrator's decision is $d((x_1, \emptyset), \emptyset) = 1$; (iii) the potential evidence is (x_1, y_0), the arbitrator's decision is $d((x_1, \emptyset), \emptyset) = 1$ and the defendant did not obtain the complete evidence, which occurs with probability $1 - e_D\theta$.

At equilibrium, the parties' conjectures are correct and effort levels are mutual best-responses given the arbitrator's strategy. There are then two possibilities.

[9] These claims may need to be qualified when cross-examination is introduced.

(i) Passive Defendant equilibrium. When the arbitrator's adjudication strategy is $d((x_1, \emptyset), \emptyset) = 0$, the conditional probability that the plaintiff succeeds is

$$[P(x_1, y_1) + P(x_0, y_1)]\theta.$$

The defendant gains nothing from gathering evidence while the plaintiff's investigation effort solves the first-order condition

$$C'(e_P) = [P(x_1, y_1) + P(x_0, y_1)]\theta. \tag{6}$$

The solution is interior because of the 'Inada condition' on investigation costs. The investigation efforts in the *pd*-equilibrium are denoted by e_P^{pd} and e_D^{pd}.

(ii) Active Defendant equilibrium. When the arbitrator's strategy is $d((x_1, \emptyset), \emptyset) = 1$, the conditional probability that the plaintiff succeeds is

$$P(x_0, y_1)\theta + P(x_1, y_1) + P(x_1, y_0)(1 - e_D\theta).$$

Substituting in the payoff functions, gathering evidence is now profitable for both parties. The equilibrium investigation efforts solve the system of first-order conditions:

$$\begin{aligned} C'(e_P) &= P(x_0, y_1)\theta + P(x_1, y_1) + P(x_1, y_0)(1 - e_D\theta), && (7) \\ C'(e_D) &= e_P P(x_1, y_0)\theta. && (8) \end{aligned}$$

We denote by e_P^{ad} and e_D^{ad} the investigation efforts in the *ad*-equilibrium.

Lemma 2 *When these equilibria exist, (i) there is a unique Passive Defendant equilibrium with* $e_P^{pd} > e_D^{pd} = 0$*; (ii) there is a unique Active Defendant equilibrium with* $e_P^{ad} > e_D^{ad} > 0$*.*

For the *pd*-equilibrium, uniqueness follows trivially from (6) and the convexity of the investigation cost. For the *ad*-equilibrium, (7) and (8) imply that the defendant's best response investigation effort is increasing in e_p and the plaintiff's is decreasing in e_D. The solution is therefore unique. Observe that in both cases the plaintiff is more likely to be the better informed party.

Arbitrator's beliefs. We now consider the conditions for the arbitrator's beliefs to support the strategy underlying either the Passive or Active Defendant equilibria. From the foregoing, the critical belief is the one associated with the decision $d((x_1, \emptyset), \emptyset)$ when the plaintiff submits partial evidence and the defendant remains silent. The arbitrator then weighs the possibility that the plaintiff observed the complete evidence against the possibility that the defendant also did. The first consideration induces skepticism vis-à-vis the plaintiff, the second tempers that skepticism. The resulting belief depends on the arbitrator's conjecture of the defendant's effort in gathering evidence. To make this explicit, we write the belief as $\beta((x_1, \emptyset), \emptyset; e_D)$ where e_D is the arbitrator's conjecture.

Lemma 3 *$\beta((x_1, \emptyset), \emptyset; e_D) > \frac{1}{2}$ if and only if*

$$k_P(1-\theta) > k_D(1-\theta e_D) \tag{9}$$

where

$$k_P \equiv pqh - (1-p)(1-q)(1-h) > k_D \equiv (1-p)(1-q)h - pq(1-h).$$

Condition (9) holds for all $e_D \geq 0$ if $\theta < \theta_a \equiv 1 - (k_D/k_P)$ and otherwise if $e_D > \varphi(\theta) \equiv (\theta - \theta_a)/[\theta(1-\theta_a)]$.

When the plaintiff submits selective evidence, the arbitrator infers that with probability θ the plaintiff observed the complete evidence and she believes that with probability θe_D the defendant also did. The left-hand side of (9) is her belief that the plaintiff did not observe the complete evidence weighted by k_P. The weight is the 'value' of finding for the plaintiff on the basis of the evidence the plaintiff would have disclosed if he could, i.e., (x_1, y_1). Specifically, k_P is the probability that ruling in the plaintiff's favor is then the correct decision minus the probability of error. Similarly, the right-hand side of (9) is the belief that the defendant did not observe the complete evidence weighted by k_D. If he could, the defendant would have disclosed (x_1, y_0). The weight is the probability of correctly finding for the defendant on the basis of that evidence minus the probability or erroneously doing so. That $k_P > k_D$ follows from Assumption 1.

To illustrate, suppose $e_D = 1$ even though this will never occur. The condition (9) is then always satisfied. By submitting $(x_1, \emptyset)$ the plaintiff reveals that he had access to some evidence. With $e_D = 1$, the arbitrator believes that the defendant also did. Skepticism towards the plaintiff is then perfectly counterbalanced by skepticism towards the defendant. It follows that the arbitrator's belief is the naïve posterior $P(\omega_1 \mid x_1) > \frac{1}{2}$, hence she finds for the plaintiff. At the other extreme, when $e_D = 0$, condition (9) can only be satisfied for a sufficiently small θ. The arbitrator leans towards the plaintiff only if the probability that he might have observed the complete evidence is below some critical value. For any given θ, the lemma defines the lower bound on e_D for the arbitrator to find for the plaintiff when $(x_1, \emptyset)$ is submitted and the defendant is silent.

Equilibria. We can now put the preceding results together and state our first main proposition.

Proposition 1 *An equilibrium with $\beta(\emptyset, \emptyset) < \frac{1}{2}$ always exists. When $\theta < \theta_a$, it is the Active Defendant equilibrium. When $\theta \geq \theta_a$, it is either the Passive Defendant equilibrium which then always exists or the Active Defendant equilibrium which also exists provided θ is not too large.*

Figure 2 provides an illustration. Let $e_D^{ad}(\theta)$ be the defendant's investigation effort in the solution to the system (7) and (8), hence $e_D^{ad}(0) = 0$ and $e_D^{ad}(\theta) < 1$ for all θ. From Lemma 3, $\varphi(\theta)$ is an increasing concave function with $\varphi(\theta_a) = 0$ and $\varphi(1) = 1$. The curves $e_D^{ad}(\theta)$ and $\varphi(\theta)$ therefore intersect, which occurs at $\theta = \theta_b$ in the figure. When $\theta \geq \theta_b$, the unique outcome is the *pd*-equilibrium because the conditions of Lemma 3 do not hold. When $\theta < \theta_a$, by contrast, these conditions always hold and the unique outcome is the *ad*-equilibrium. When $\theta \in [\theta_a, \theta_b)$, both types of equilibria exist. In the *ad*-equilibrium, when the plaintiff submits partial evidence and the defendant is silent, the arbitrator adjudicates in the plaintiff's favor because $e_D^{ad}(\theta) > \varphi(\theta)$. In the *pd*-equilibrium, the arbitrator adjudicates against the plaintiff because $e_D^{pd} = 0 < \varphi(\theta)$.[10]

[10] In general, one cannot rule out that the curves intersect more than onces. However, there always exists $\theta_b < 1$ such that only the PD equilibrium remains when $\theta > \theta_b$.

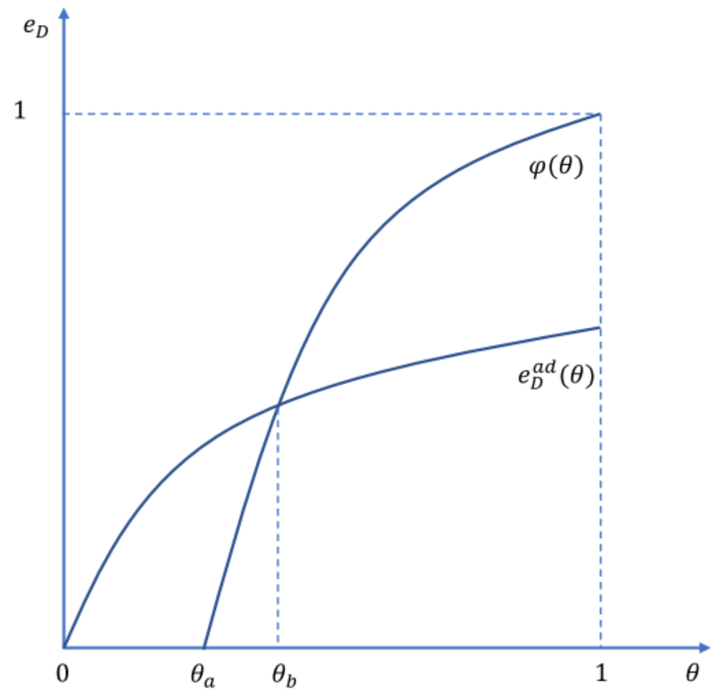

Figure 2. Active and Passive Defendant Equilibria

Accuracy in adjudication. This is measured by the probability of correct adjudication, equivalently by the arbitrator's expected utility

$$\begin{aligned}\overline{u} &= p\Pr(d=1 \mid \omega_1) + (1-p)\Pr(d=0 \mid \omega_0) \\ &= 1-p+[p\Pr(d=1 \mid \omega_1) - (1-p)\Pr(d=1 \mid \omega_0)]\,. \qquad (10)\end{aligned}$$

where $\Pr(d \mid \omega)$ denotes the probability of decision d at equilibrium, conditional on the true state being ω.

In the Passive Defendant equilibrium,

$$\overline{u}^{pd} = 1-p+e_P^{pd}\Delta^{pd} \qquad (11)$$

where

$$\Delta^{pd} = \theta\,[ph-(1-p)(1-h)]\,. \qquad (12)$$

In equation (11) the term $1-p$ is the probability of correct adjudication merely on the basis of the primary burden of proof assignment, i.e., it is the probability of the arbitrator's default decision being the right one when no evidence is communicated.[11] The next term is the value added by the communication phase. This depends on the plaintiff's investigation effort and the informativeness of the communication phase. Evidence matters only when it yields a decision in favor of the plaintiff, i.e., reversing the default decision under no evidence. Δ^{pd} is then the improvement in decision-making that results from the communication stage. It is the probability of correctly finding for the plaintiff minus the probability of erroneously doing so.

In the Active Defendant equilibrium,

$$\overline{u}^{ad} = 1 - p + e_P^{ad}\Delta^{ad} \tag{13}$$

where

$$\Delta^{ad} = \Delta^{pd} + \left[k_P(1-\theta) - k_D(1-\theta e_D^{ad})\right], \tag{14}$$

see the proof of the next proposition. From Lemma 3, the expression in brackets is positive whenever the *ad*-equilibrium exists.

Proposition 2 *When both the Passive and Active Defendant equilibria exist, the latter yields a smaller probability of error. The plaintiff more often submits evidence,* $e_P^{ad} > e_P^{pd}$*, and the communication stage is more informative,* $\Delta^{ad} > \Delta^{pd}$*.*

The Active Defendant equilibrium yields a smaller error because a 'productive' communication phase is reached more often and because the defendant now also investigates, improving the arbitrator's information and inferences at the communication phase. The defendant's investment in gathering evidence improves decision-making by allowing the plaintiff to succeed when the only evidence submitted is $(x_1, \emptyset)$. This decision is now sequentially rational for the arbitrator because it is sufficiently likely that the defendant could have provided counterevidence, hence the arbitrator is justified in being less skeptical vis-à-vis the plaintiff.[12] In addition, because he faces a

[11] Should no evidence be forthcoming, assigning the burden to the plaintiff yields the correct decision with probability $1-p$. This is the appropriate assignment because $p \le \frac{1}{2}$.

[12] Decision-making is not improved when only the defendant submits evidence, i.e., $m_P = \emptyset$ and $m_D = (x_1, y_0)$, because the decision would have been against the plaintiff even in the absence of such evidence.

greater probability of prevailing, the plaintiff invests more in gathering evidence. In the sequel, when both types of equilibria exist, we select the more informative 'arbitrator-preferred' equilibrium.[13]

4 Cross-Examination

A report can be deceitful through withholding of facts. In our set-up, this can arise in the Active Defendant equilibrium when the plaintiff reports $(x_1, \emptyset)$. The plaintiff then prevails if the defendant is unable to provide counterevidence. We view cross-examination as an action on the part of the defendant that attempts to weaken – but could also turn out to strengthen – the probative value of the plaintiff's submission. The 'cross-examination test' allows the arbitrator to update her belief that the plaintiff reported the 'whole truth'.

At the investigation stage, the defendant can now invest both to gather direct evidence and to uncover the means of effective cross-examination. In judicial procedures, during the course of a trial, cross-examination is often conducted through a sequence of so-called 'leading questions' which the witness can only answer with 'yes' or 'no'. We model cross-examination as a set of 'questions' put to the plaintiff's submission and which yields what will be referred to as ancillary evidence. The defendant uncovers the appropriate questions with probability η_D at a cost $K(\eta_D)$ at the investigation stage, an increasing convex function with $K(0) = K'(0) = 0$ and $K'(1) \geq 1$.

Ancillary evidence. Let A be the event "plaintiff obtained evidence" and B the sub-event "plaintiff observed the complete evidence" with $\overline{B}$ its complement in A. Consistent with our assumptions so far, $\Pr(B \mid A) = \theta$. Let $z \in \{0, 1\}$ be a variable correlated with how well informed the plaintiff is but otherwise independent of the underlying facts of the dispute. We assume

$$\Pr(z = 1 \mid B) = \Pr(z = 0 \mid \overline{B}) = v, \quad \text{where } v \in (\tfrac{1}{2}, 1).$$

[13] The arbitrator could announce at the outset that she will be satisfied with the evidence $(x_1, \emptyset)$ for ruling in the plaintiff's favor, allowing coordination on the better outcome. The argument is the same as for the assignment of the burden of proof; see footnote 3 .

Cross-examination raises issues with the plaintiff's submission, the outcome of which is to allow the arbitrator to observe the realization of z. Applying Bayes' rule,

$$\hat{\theta}_1 \equiv \Pr(B \mid A \text{ and } z = 1) = \frac{\theta v}{\theta v + (1-\theta)(1-v)} \tag{15}$$

$$\hat{\theta}_0 \equiv \Pr(B \mid A \text{ and } z = 0) = \frac{\theta(1-v)}{\theta(1-v) + (1-\theta)v} \tag{16}$$

hence $\hat{\theta}_0 < \theta < \hat{\theta}_1$. Thus, cross-examination allows the arbitrator to revise her beliefs that the plaintiff withheld evidence.

When $z = 1$, the plaintiff fails the cross-examination test and the probative value of $(x_1, \emptyset)$ is weakened. The arbitrator then adopts a more skeptical stance towards the plaintiff because greater weight is put on the possibility of strategic manipulation. Conversely, $z = 0$ strengthens the plaintiff's case. Borrowing from statistical terminology, we refer to z as *ancillary* evidence by contrast with the *direct* evidence which is intrinsically informative about the issue at stake.[14]

The time-line is now modified by adding a cross-examination stage in the communication phase. At stage 1, the parties invest in the gathering of direct evidence; the defendant also simultaneously invests in uncovering leading questions. The parties' investigation efforts and their outcome remains private information. At stage 2, the parties simultaneously decide what direct evidence to disclose. At stage 3, depending on the outcome of the previous stage, the defendant decides whether to cross-examine the plaintiff (if he can). At stage 4, the arbitrator updates her beliefs on the basis of the direct and ancillary evidence and adjudicates.

Although the procedure now allows cross-examination, at equilibrium the defendant may actually not invest in acquiring the means of cross-examination. The outcome will then again be either the Passive or the Active

[14] An ancillary statistic does not depend on the unknown parameter to be estimated but nevertheless provides valuable information (e.g., the sample size as opposed to the sample mean in estimating the first moment of a distribution). Conlon (2009) and Sinclair-Desgagné (2009) remark that, although the terminology is not used, the usefulness of 'ancillary signals' is ubiquitous in the principal-agent model, e.g., other agents' performance levels in a multi-agent context.

Defendant equilibrium as in the preceding section, but with the added strategy $\eta_D = 0$, i.e., the defendant does not invest in cross-examination. When $\eta_D > 0$ at equilibrium, the outcome is called an Active Cross-Examination equilibrium, referred to as an *ac*-equilibrium for short.

Strategies and payoffs. The defendant's cross-examination strategy in the communication phase is denoted by χ_D. When he does not cross-examine, we write $\chi_D = \emptyset$; when he does, we write $\chi_D = z$ because the arbitrator then observes the realization of z. The adjudication strategy is $d(m_P, m_D, \chi_D)$ where the first two arguments denote as before the direct evidence submitted. Similarly, the arbitrator's belief is denoted $\beta(m_P, m_D, \chi_D)$.

At stage 2, the parties' strategies concerning the submission of direct evidence remain the same. The arbitrator's decision when the direct evidence reduces to $m_P = (x_1, \emptyset)$ is again pivotal. Our first step is to characterize the defendant's strategy as a function of this decision.

Case 1: $d((x_1, \emptyset), \emptyset, \emptyset) = 0$

The plaintiff then loses in the absence of cross-examination. Hence the defendant gains nothing by cross-examining. Indeed, this could be detrimental if it showed that the plaintiff most likely did not suppress evidence. Therefore, if the Passive Defendant equilibrium is the unique outcome when cross-examination is not allowed, it is also the unique outcome when the procedure allows it, i.e, the defendant does not search for direct evidence nor for the means of cross-examination.

Case 2: $d((x_1, \emptyset), \emptyset, \emptyset) = 1$

The plaintiff now prevails in the absence of cross-examination. Hence the defendant may possibly gain from cross-examination. The adjudication decision following cross-examination is $d((x_1, \emptyset), \emptyset, z)$. When $d((x_1, \emptyset), \emptyset, 1)) = 1$, the plaintiff succeeds even if he fails the cross-examination test. At the investigation stage, therefore, the defendant does not invest in acquiring the means of cross-examination and the outcome is the same as in the Active Defendant equilibrium. This case is of limited interest and can arise only because cross-examination is insufficiently informative. Conversely, when the adjudication decision is $d((x_1, \emptyset), \emptyset, 1)) = 0$, the plaintiff loses if he fails the cross-examination test. The defendant's strategy is therefore to cross-examine when he has no direct evidence to counter the plaintiff's submission. Investing in acquiring the means of cross-examination is now profitable, i.e., we have an Active Cross-Examination equilibrium.

In this equilibrium, the defendant's expected payoff at the investigation stage is

$$\pi_D = -\,e_P S(e_D, \eta_D) - C(e_D) - K(\eta_D) \tag{17}$$

where

$$\begin{aligned} S(e_D, \eta_D) \;\equiv\;& P(x_0, y_1)\theta + P(x_1, y_1)[1 - \eta_D(1-\theta)(1-v)] \\ &+ P(x_1, y_0)(1 - \theta e_D)[1 - \eta_D(\theta v + (1-\theta)(1-v))] \end{aligned} \tag{18}$$

is the probability that the plaintiff succeeds conditional on obtaining evidence. In (17), e_P is the defendant's deterministic conjecture of the plaintiff's investigation effort.

To decipher (18), observe first that, when the potential evidence is (x_1, y_1), the plaintiff submits $(x_1, \emptyset)$ only when he does not observe the complete evidence. This occurs with probability $1 - \theta$. The plaintiff is then cross-examined with probability η_D and the outcome is $z = 1$ with probability $1 - v$. When the potential evidence is (x_1, y_0), the plaintiff submits only partial evidence whether fully informed or not. He is therefore cross-examined if the defendant found the means to do so and did not obtain direct counterevidence. The probability of $z = 1$ conditional on cross-examination is then $\theta v + (1-\theta)(1-v)$. Finally, it is implicit that the plaintiff prevails when the outcome of cross-examination is $z = 0$.

The defendant's expected payoff need not be concave in his decision variables e_D and η_D. This raises the possibility that, for a given conjecture of the plaintiff's investigation effort, the defendant's best-response is multi-valued. For instance, he may be indifferent between investing substantially at the investigation stage, thereby allowing the plaintiff only a small probability of prevailing, or conversely economizing on investigation expenditures and allowing the plaintiff a better prospect. For the time being, in order to simplify the exposition, we introduce a technical assumption ensuring that the defendant's best response is unique. We relax this condition in Section 5 and consider the possibility that an Active Cross-Examination equilibrium may require that the defendant plays a mixed strategy.[15]

Assumption 2: For all θ consistent with an Active Defendant equilibrium and all e_P, $\eta_D \in [0,1]$,

$$\frac{C''(e_D)}{C'(e_D)}\frac{K''(\eta_D)}{K'(\eta_D)} \geq \frac{\theta[\theta v + (1-\theta)(1-v)}{(1-\theta)[1-\theta v-(1-\theta)(1-v)]} \tag{19}$$

Recall that an *ad*-equilibrium exists only if θ is not too close to unity, implying that the right-hand side of (19) is not arbitrarily large. Assumption 2 is a 'sufficient convexity condition' with respect to the defendant's cost functions in gathering direct evidence and in searching for the means of cross-examination. The assumption ensures that the defendant's payoff function (17) is strictly quasiconcave, hence his best response his unique.

Accordingly, we may write the plaintiff's payoff as

$$\pi_P = e_P S(e_D, \eta_D) - C(e_P) \tag{20}$$

where e_D and η_D are the plaintiff's conjectures of the defendant's effort levels.

Let e_P^{ac} denote the plaintiff's investigation effort in an Active Cross-Examination equilibrium and let (e_D^{ac}, η_D^{ac}) be the defendant's strategy. Then

$$e_P^{ac} \in \arg\max_{e_P} e_P S(e_D^{ac}, \eta_D^{ac})] - C(e_P), \tag{21}$$

$$(e_D^{ac}, \eta_D^{ac}) \in \arg\min_{e_D,\eta_D} e_P^{ac} S(e_D, \eta_D) + C(e_D) + K(\eta_D). \tag{22}$$

This yields the following.[16]

Lemma 4 *When it exists, there is a unique Active Cross-Examination equilibrium with $\eta_D^{ac} > 0$ and $e_P^{ac} > e_D^{ac} > 0$.*

[15] The main results are then essentially unchanged but the exposition is more involved.

[16] To avoid repetition, the proof of the implications of Assumption 2 are given in the proof of Lemma 4, but otherwise the Appendix provides proofs of the henceforth remaining claims (or their reformulation) only for the general case where Assumption 2 is dropped and the equilibrium may involve mixed strategies. See Section 5.

In the *ac*-equilibrium, the defendant invests on both margins, i.e., in acquiring direct evidence and in acquiring means of cross-examination. This follows trivially from the Inada conditions on cost functions. Observe that, as in the Passive and Active Defendant equilibria, the plaintiff is more likely to be the better informed party.

Equilibria when cross-examination is allowed. We first determine the conditions for the arbitrator's beliefs to be consistent with an Active Cross-Examination equilibrium. From the preceding discussion, the beliefs must support the decision $d((x_1, \emptyset), \emptyset, \emptyset) = 1$, i.e., the plaintiff prevails when he reports $(x_1, \emptyset)$ and is not cross-examined, and the decision $d((x_1, \emptyset), \emptyset, 1)) = 0$, i.e., the plaintiff loses if he is cross-examined and fails the cross-examination test. As before, the arbitrator's belief will depend on her conjecture of the defendant's effort in gathering direct evidence. We make this explicit by writing the beliefs as $\beta((x_1, \emptyset), \emptyset, \emptyset; e_D)$ and $\beta((x_1, \emptyset), \emptyset, 1; e_D)$ respectively.

Lemma 5 $\beta((x_1, \emptyset), \emptyset, 1; e_D) \leq \frac{1}{2} < \beta((x_1, \emptyset), \emptyset, \emptyset; e_D)$ *if and only if*

$$k_P(1-\theta) > k_D(1-\theta e_D) \geq k_P(1-\hat{\theta}_1) \tag{23}$$

The first inequality in (23) is similar to condition (9) in Lemma 3. It ensures that the plaintiff prevails if the only evidence submitted is $(x_1, \emptyset)$ and there is no cross-examination. The second inequality in (23) is a similar condition but in terms of the posterior belief conditional on the cross-examination outcome $z = 1$. It ensures that the plaintiff loses when he fails the cross-examination test. In either case, the arbitrator takes into account the possibility that both the plaintiff and the defendant withheld evidence, i.e., the defendant may himself be deceitful when he chooses to cross-examines rather than disclose direct evidence.[17]

The inequalities in (23) can be simultaneously satisfied only if cross-examination is sufficiently informative, i.e., the updated $\hat{\theta}_1$ must be sufficiently above the prior θ. This requires that v be sufficiently large. We

[17] Condition (23) insures that the defendant prevails if he passes the cross-examination test. The requirement is $k_P(1-\hat{\theta}_0) > k_D(1-\theta e_D)$ which follows trivially from the first inequality in (23) because $\hat{\theta}_0 < \theta$.

assume that this is always the case, thus ruling out the possibility that allowing cross-examination has no effect when the Active Defendant equilibrium exists.

Proposition 3 *Suppose the procedure without the right of cross-examination yields the Active Defendant equilibrium. Then allowing cross-examination reduces the defendant's investment in gathering direct evidence. The outcome is either (i) the Active Cross-Examination equilibrium with* η_D^{ac}*,* $e_D^{ac} > 0$ *and where* $e_D^{ac} < e_D^{ad}$*; or (ii) the unique outcome is the Passive Defendant equilibrium with* $\eta_D^{pd} = e_D^{pd} = 0$*.*

For the defendant, direct evidence to counter the plaintiff or effective cross-examination are substitutes. Therefore, when cross-examination is allowed, the defendant invests less in the gathering of direct evidence. In the Active Cross-examination outcome, he compensates to some extent by searching for the means of cross-examination. However, it may also be that allowing cross-examination yields the Passive Defendant equilibrium even though the procedure without the right of cross-examination would result in the Active Defendant equilibrium.

The possibilities are illustrated in Figure 3. Let $e_D^{ac}(\theta)$ denote the defendant's effort at gathering direct evidence in the solution of the investment game defined by (21) and (22). The other curves are as in Figure 2. In the procedure without the right of cross-examination, the Active Defendant equilibrium exists for $\theta < \theta_b$. From Lemma 5, the strategy yielding $e_D^{ac}(\theta)$ is part of an Active Cross-Examination equilibrium only when the first inequality in (23) holds. Borrowing from Lemma 3, this amounts to the condition $e_D^{ac}(\theta) < \varphi(\theta)$. Moreover, we know from Proposition 3 that $e_D^{ac}(\theta) < e_D^{ad}(\theta)$. Therefore $e_D^{ac}(\theta)$ crosses $\varphi(\theta)$ at some $\theta_c < \theta_b$ as shown. In the interval $[\theta_c, \theta_b)$, an *ad*-equilibrium exists in the procedure without the right of cross-examination (together with the *pd*-equilibrium). However, the unique outcome when cross-examination is allowed is the *pd*-equilibrium because condition (23) in Lemma 5 then does not hold. Intuitively, the defendant's effort in gathering direct evidence in the Active Cross-Examination investment game would be too small. It would induce the arbitrator to adopt a sufficiently skeptical stance to find against the plaintiff when he submits partial evidence and the defendant is silent. Hence the defendant has in fact no need to search for evidence or the means of cross-examination.

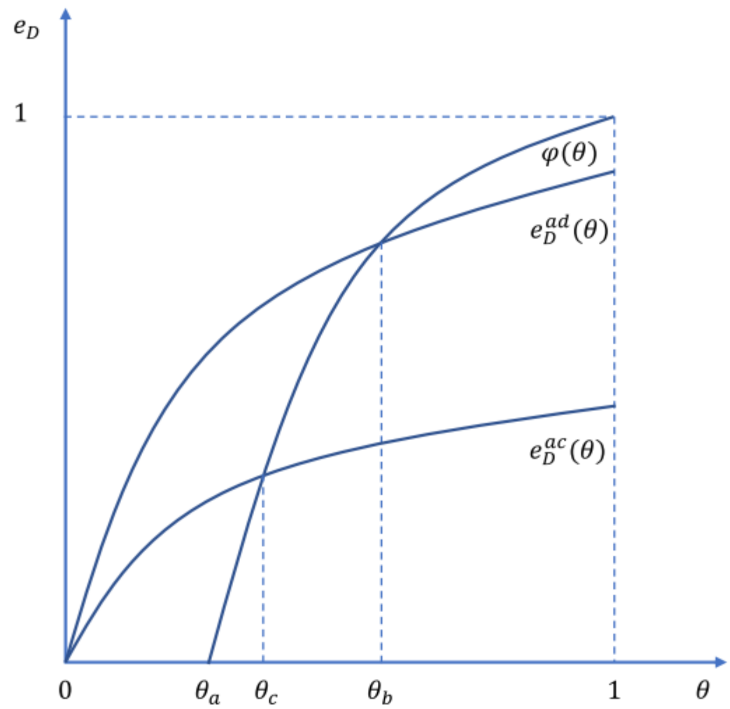

Figure 3. Equilibria with the Right of Cross-Examination

Accuracy in adjudication. Using the same approach as in Section 3, the probability of correct adjudication under the procedure allowing cross-examination is

$$\overline{u} = 1 - p + e_P \Delta \tag{24}$$

where e_P is the plaintiff's investigation effort at equilibrium and Δ is the informational value of the communication phase. When the outcome is the pd-equilibrium, the plaintiff's investigation effort is e_P^{pd} and the communication phase is worth Δ^{pd}. When the outcome is an ac-equilibrium, the plaintiff's effort is e_P^{ac} and the communication phase can be shown to be worth[18]

$$\begin{aligned}\Delta^{ac} &= \Delta^{pd} + k_P(1-\theta)[1-(1-v)\eta_D^{ac}] \\ &\quad - k_D[(1-\theta e_D^{ac})(1-(\theta v + (1-\theta)(1-v))\eta_D^{ac})]. \end{aligned} \tag{25}$$

Equivalently (see the proof of the next proposition), the value of the communication phase can be rewritten as

$$\begin{aligned}\Delta^{ac} &= \Delta^{pd} + (1-\eta_D^{ac})[k_P(1-\theta) - k_D(1-\theta e_D^{ac})] \\ &\quad + \eta_D^{ac}[1-\theta v - (1-\theta)(1-v)][k_P(1-\hat{\theta}_0) - k_D(1-\theta e_D^{ac})]. \end{aligned} \tag{26}$$

From Lemma 5, the expression in the first pair of square brackets is positive when the *ac*-equilibrium exists. The expression in the second pair of brackets is also positive because $\hat{\theta}_0 < \theta$, where $\hat{\theta}_0$ is the up-dated probability that the plaintiff observed the complete evidence when he passes the cross-examination test.

The next proposition compares the quality of adjudication under the procedures with and without the right of cross-examination.

Proposition 4 *Suppose the procedure without the right of cross-examination yields the Active Defendant equilibrium. Then, when cross-examination is allowed, the plaintiff submits evidence less often, $e_P < e_P^{ad}$, or the communication phase is less informative, $\Delta < \Delta^{ad}$, or both.*

When the procedure allowing cross-examination results in the Passive Defendant outcome, accuracy unambiguously decreases because both inequalities in the proposition simultaneously hold. In the Active Cross-Examination outcome, by contrast, a possible consequence of allowing cross-examination is to improve the informativeness of the communication phase, i.e., $\Delta^{ac} > \Delta^{ad}$. The trade-off is then that this necessarily chills the plaintiff's investigation efforts, i.e., $e_P^{ac} < e_P^{ad}$, so that a 'productive' communication phase is reached less often. The plaintiff investigates less because more informative communication reduces his chances of prevailing conditional on having access to evidence. The effect on the quality of adjudication

[18] Note that the expression has the same form as (14) when η_D is set equal to zero.

is therefore in general ambiguous. It is easy to provide numerical examples where allowing cross-examination either improves or worsens adjudication. We next discuss the possibility that the communication phase in an Active Cross-Examination equilibrium is itself less informative than in the Active Defendant equilibrium.

The defendant's effort mix. It may be that in the *ac*-equilibrium both $\Delta^{ac} < \Delta^{ad}$ and $e_P^{ac} < e_P^{ad}$. To see why this can arise, it is convenient to decompose the defendant's problem at the investigation stage as first minimizing the cost of reaching some probability s that the plaintiff prevails (in the feasible range of such probabilities) and then choosing the probability. The first step yields the cost function

$$G(s) \equiv \min_{e_D, \eta_D \geq 0} C(e_D) + K(\eta_D) \text{ s.t. } S(e_D, \eta_D) \leq s. \tag{27}$$

Under a 'sufficient convexity' condition, the cost minimizing pair of investigation efforts is unique and $G(s)$ is a twice differentiable decreasing convex function.

The parties' expected payoffs at the investigation stage can now be written as $\pi_P = se_P - C(e_P)$ and $\pi_D = -\,se_P - G(s)$. The plaintiff chooses e_P and the defendant chooses s. The outcome in the Active Cross-Examination investment game is therefore e_P^{ac} and s^{ac} solving

$$C'(e_P) \quad = \quad s, \tag{28}$$

$$-\,G'(s) \quad = \quad e_P. \tag{29}$$

Figure 4 depicts the equilibrium.

We now compare this outcome with the Active Defendant equilibrium when cross-examination is not allowed. In this case the defendant is constrained to $\eta_D = 0$. Let $\hat{e}_D(s)$ be implicitly defined by $S(\hat{e}_D(s), 0) = s$. As a function of the probability s that he chooses to implement, the defendant's expected payoff is now $\pi_D = -\,se_P - C(\hat{e}_D(s))$. The outcome of the Active Defendant investment game is the pair e_P^{ad} and s^{ad} solving (28) together with

$$-\frac{C'(\hat{e}_D(s))}{S_{e_D}(\hat{e}_D(s), 0)} = e_P. \tag{30}$$

The left-hand side is the marginal cost of reducing the plaintiff's probability of prevailing when this can only be done by gathering direct evidence. Under a 'sufficient convexity' condition, the defendant's marginal cost of reducing the plaintiff's probability of success is larger when cross-examination is not allowed. Figure 4 depicts the resulting equilibrium. In the 'typical' outcome illustrated in the figure, $s^{ac} < s^{ad}$ and $e_P^{ac} < e_P^{ad}$.

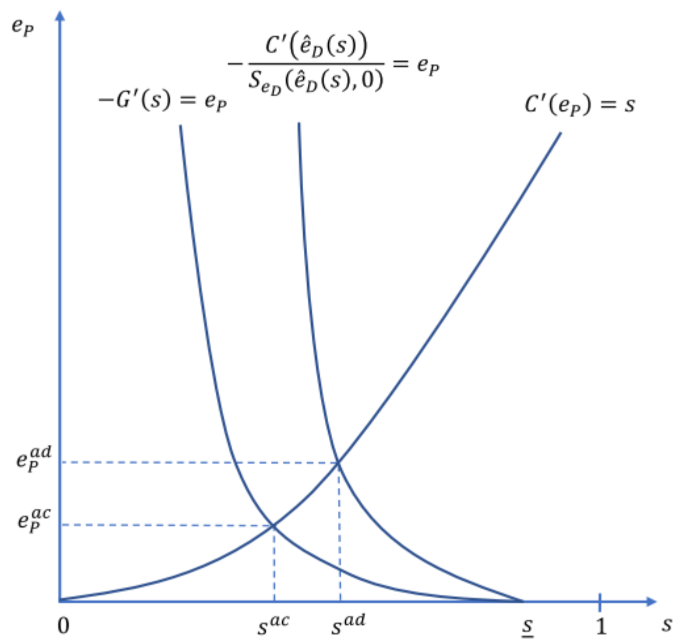

Figure 4. $e_P^{ac} < e_P^{ad}$ in an Active Cross-Examination Equilibrium

Next we discuss some features of these equilibria with respect to the defendant's effort mix. Figure 5 shows iso-value contours in the (e_D, η_D) plane. The thick curve is an iso-expenditure contour defined by

$$C(e_D) + K(\eta_D) = G(s^{ac}),$$

i.e., it is the locus of e_D and η_D pairs yielding the same investment expenditure for the defendant as in the ac-equilibrium. The thinner curves are iso-probability contours, defined as pairs of e_D and η_D yielding the same conditional probability of success to the the plaintiff. One contour is $S(e_D, \eta_D) = s^{ac}$, the other is $S(e_D, \eta_D) = s^{ad}$. The defendant's expenditure decreases as we move downwards to lower level iso-expenditure contours. By contrast, the probability that the plaintiff prevails decreases as we move upwards to higher iso-probability contours. Both types of curve are easily shown to be concave to the origin. In the ac-equilibrium, e_D^{ac} and η_D^{ac} are at the tangency of the iso-expenditure and iso-probability contours at the s^{ac} level (shown as point E in the figure). In the ad-equilibrium, we have a corner solution because η_D is constrained to zero. Consistent with Proposition 3, $e_D^{ac} < e_D^{ad}$. Consistent with Figure 4, $s^{ac} < s^{ad}$.

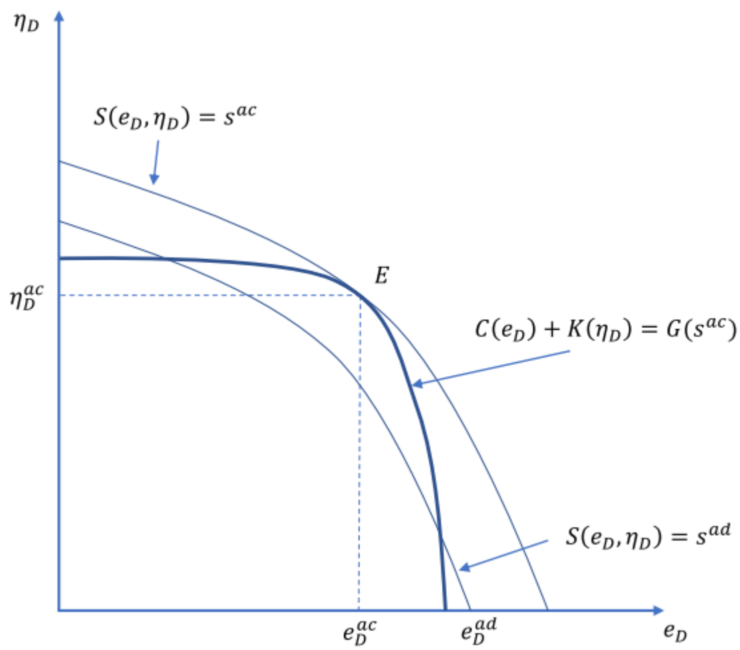

Figure 5. The Defendant's Effort Mix in an Equilibrium with $s^{ac} < s^{ad}$

Figure 6 adds iso-information contours. Borrowing from (26), an iso-information contour is defined by the constancy of

$$\begin{aligned}\Delta(e_D, \eta_D) \quad \equiv \quad & \Delta^{pd} + k_P(1-\theta)[1-(1-v)\eta_D] \\ & - k_D(1-\theta e_D)[1-(\theta v + (1-\theta)(1-v))\eta_D]. \qquad (31)\end{aligned}$$

For e_D in an appropriate range, $\Delta(e_D, \eta_D)$ is the informational value of the communication phase as a function of the defendant's investigation efforts.[19] Iso-information contours are also concave to the origin. From equation (14), $\Delta(e_D^{ad}, 0) = \Delta^{ad}$. From equation (25), $\Delta(e_D^{ac}, \eta_D^{ac}) = \Delta^{ac}$. The two corresponding loci are shown as dotted curves. A higher-level locus means a more informative communication phase.

As shown, $\Delta^{ac} < \Delta^{ad}$. Because $e_P^{ac} < e_P^{ad}$ as well, allowing cross-examination is therefore unambiguously detrimental. The possibility arises because iso-information contours are steeper than iso-probability loci. Specifically, from (18) and (25), it it easily verified that

$$-\left.\frac{d\eta_D}{de_D}\right|_{\Delta(e_D,\eta_D)=\mathrm{ct}} > -\left.\frac{d\eta_D}{de_D}\right|_{S(e_D,\eta_D)=\mathrm{ct}} \quad \text{for all } v < 1.$$

In the limiting case where cross-examination is perfectly informative (i.e., $v = 1$), iso-probability and iso-information loci overlap at comparable levels. The defendant's interests are then perfectly aligned with the arbitrator's interests so to speak. Reducing the probability that the plaintiff prevails is equivalent to maximizing the information value of the communication phase. Moreover, given his level of expenditure at the investigation stage, the defendant chooses the effort mix, between gathering direct evidence and obtaining the means of cross-examination, that the arbitrator would want him to choose.

This is no longer true when cross-examination is noisy (although informative), as assumed here. Noisy cross-examination allows for the possibility depicted in Figure 6. In any case, there is now always a wedge between the defendant's and the arbitrator's interests which shows up in the defendant's effort mix. While the defendant chooses (e_D^{ac}, η_D^{ac}), for the same level of expenditure the arbitrator would want him to exert more effort on gathering direct evidence and less on cross-examination. This is an immediate consequence of the single-crossing property between iso-probability and iso-information contours. In Figure 6, effort pairs to the right of point E on the ac-level iso-expenditure contour yield a more informative communication

[19] This interpretation may not hold for e_D arbitrarily small but is does for $e_D \in [e_D^{ac}, e_D^{ad}]$, which is what matters.

phase.

Corollary 1 *From the arbitrator's point of view, the defendant's effort mix is distorted towards too much investment in cross-examination at the expense of too little gathering of direct evidence.*

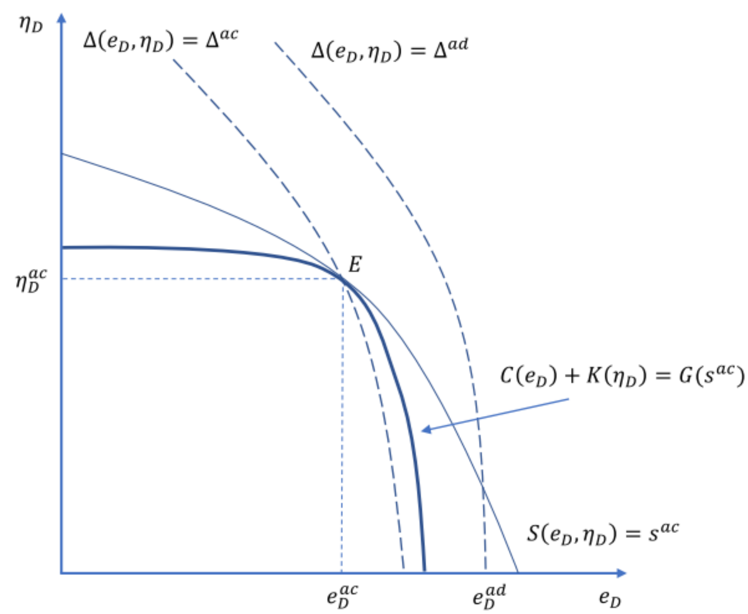

Figure 6. A Case where $e_P^{ac} < e_p^{ad}$ and $\Delta^{ac} < \Delta^{ad}$

5 Discussion

Deterring versus provocative evidence gathering. In a contest model where the probability of winning depends on the parties' expenditure, Katz (1988) obtains that the expenditure of the ex ante favored party is 'deterring' while that of the underdog is 'provocative'. We have a similar result if the underdog is interpreted as the party with the burden of proof, the plaintiff in our setting. From the parties' reaction functions, greater investment in gathering evidence by the plaintiff provokes the defendant to invest more in gathering evidence and in acquiring means of cross-examination when the procedure allows it. Conversely, greater investment on the part of the defendant reduces the plaintiff's incentives to investigate.[20]

This contrasts with Kartik et al. (2017). They study the case where the parties' preferences are linear in the decison maker's posterior belief. Efforts in acquiring information are then strategic substitutes. By contrast, in our setting the adjudicator makes a binary decision, she must find for the plaintiff or the defendant. The parties' payoffs are then discontinuous functions of the adjudicator's posterior belief. One party wants to push the belief above the threshold of one half, the other party wants to push the belief below the threshold. Another implication is that in Kartik et al. the decision maker may be better off with only one sender, i.e., competition between opposed parties may be detrimental. This cannot arise in our setting. For instance, the Active Defendant outcome is always more informative than the Passive Defendant outcome.[21]

Analogy with mandatory disclosure. There is a similarity between the disincentive effect of cross-examination and the well-known consequences of mandatory disclosure rules when information is endogenous. In some one-sender settings (Matthews and Postelwaite 1985, Shavell 1994, Schweizer 2017) and similarly in the multi-sender setting of Kartik at al. (2017), mandatory disclosure completely eliminates incentives to acquire information. Again this is related to the continuity of the sender's payoff with respect to the decision maker's belief. In our setting, the plaintiff would always investigate even under mandatory disclosure.[22] Nevertheless, the plaintiff would investigate less than under voluntary disclosure. The threat of cross-examination faced by the plaintiff is similar to a risk of involuntary

[20] See also Daughety and Reinganum (2000) for reaction functions exhibiting the same properties.

[21] Another difference with Kartik et al. (2017) is that, in our setting, the parties uncover multidimensional signals that are not fully independently distributed conditionally on the sate. However, these features do not explain the deterring-provocative interaction between evidence acquisition efforts. Similar reaction functions would be obtained, for instance if the plaintiff could only uncover x and the defendant could only uncover y. Of course, with unidimensional signals there would be no role for cross-examination.

[22] The defendant would also investigate for parameter values consistent with the Active Defendant equilibrium.

full disclosure, detrimental to the plaintiff. The overall effect on the quality of decision-making, however, is intricate and depends on the reactions of both parties. The defendant invests less in acquiring direct evidence because he partly substitutes towards acquiring means of cross-examination and because the plaintiff's investigation effort is chilled by the threat of cross-examination. Given that the defendant is less likely to possess evidence, the adjudicator's inferences when the plaintiff discloses only partial evidence (and is not cross-examined) are then less informed.

General case with mixed strategies. To complete the argument in Section 4, we now discard Assumption 2. A consequence is that the cost function $G(s)$ need not be convex, equivalently the defendant's expected payoff need not be concave in s. Therefore we may need to consider the possibility that the defendant plays a mixed strategy at equilibrium, randomizing between high and low values of the plaintiff's conditional probability of success. The defendant's strategy at the investigation stage is represented by the probability distribution $\sigma(e_D, \eta_D)$ over effort levels.

Let e_P^{ac} be the plaintiff's investment in an Active Cross-Examination equilibrium and let (e_D^{ac}, η_D^{ac}) denote pairs in the support of the defendant's strategy. Then, for the defendant, any pair (e_D^{ac}, η_D^{ac}) solves (22), while the plaintiff's effort solves

$$e_P^{ac} \in \arg\max_{e_P} e_P E_\sigma[S(e_D^{ac}, \eta_D^{ac})] - C(e_P) \tag{32}$$

where E_σ denotes the expected value given the probability distribution σ. Lemma 4 is modified as follows.

Lemma 4b *The Active Cross-Examination equilibrium is unique with respect to the plaintiff's investigation effort e_P^{ac}. For all e_D^{ac} and η_D^{ac} in the support of the defendant's strategy, $\eta_D^{ac} > 0$ and $e_D^{ac} > 0$. Moreover $e_P^{ac} > E_\sigma(e_D^{ac})$.*

Observe that in the *ac*-equilibrium the plaintiff is again more likely to be the better informed party.[23]

[23] This can be shown to imply $\beta(\emptyset, \emptyset) < \frac{1}{2}$. We skip the proof because it is similar to that of Proposition 1.

Consider now the arbitrator's updating when the plaintiff submits partial evidence and the defendant has no countervailing direct evidence but may cross-examine. The arbitrator's beliefs depend on his conjecture $\sigma(e_D, \eta_D)$ of the defendant's mixed strategy.

Lemma 5b $\beta((x_1, \emptyset), \emptyset, \emptyset; \sigma) > \frac{1}{2}$ *if and only if*

$$k_P(1-\theta) > k_D(1-\theta\widetilde{e}_D^{ac}) \tag{33}$$

where

$$\widetilde{e}_D^{ac} \equiv \frac{E_\sigma[e_D^{ac}(1-\eta_D^{ac})]}{E_\sigma(1-\eta_D^{ac})} \tag{34}$$

$\beta((x_1, \emptyset), \emptyset, 1; \sigma) \leq \frac{1}{2} < \beta((x_1, \emptyset), \emptyset, 0; \sigma)$ *if and only if*

$$k_P(1-\hat{\theta}_0) > k_D(1-\theta\hat{e}_D^{ac}) \geq k_P(1-\hat{\theta}_1) \tag{35}$$

where

$$\hat{e}_D^{ac} = \frac{E_\sigma(e_D^{ac}\,\eta_D^{ac})}{E_\sigma(\eta_D^{ac})} \tag{36}$$

The inequality (33) ensures that the plaintiff prevails when he submits partial evidence and is not cross-examined. When the defendant plays a mixed strategy, the arbitrator is uncertain about the defendant's investment in gathering evidence. That he does not cross-examine then provides some information. $\widetilde{e}_D^{ac}$ is the posterior probability that the defendant obtained some evidence conditional on no cross-examination. Hence $1-\theta\widetilde{e}_D^{ac}$ is the arbitrator's belief that the defendant did not have access to the complete evidence. The interpretation is otherwise the same as for Lemma 3.

The first inequality in (35) is a similar condition but in terms of the posterior belief conditional on the cross-examination outcome $z = 0$. The inequality ensures that the plaintiff prevails when he passes the cross-examination test. $\hat{\theta}_0$ is then the posterior probability that the plaintiff withheld evidence and $\hat{e}_D^{ac}$ is the posterior probability that the defendant obtained some evidence conditional on the fact that cross-examination takes place. The second inequality in (35) ensures that the plaintiff loses when $z = 1$, i.e., when he fails the cross-examination test.[24]

Proposition 3b *The claim is the same as in Proposition 3 with e_D^{ac} and η_D^{ac} interpreted as belonging to the support of σ and with $E_\sigma(e_D^{ac}) < e_D^{ad}$ instead of $e_D^{ac} < e_D^{ad}$.*

Proposition 4 remains the same.

Informed cross-examiner. We conclude this section by briefly discussing a minor change in our set-up. We assumed that the defendant did not observe the realization of z before subjecting the plaintiff to cross-examination. It is often said that a careful cross-examiner should know what to expect.[25] All of the above results hold when the defendant observes the realization of z. It suffices to change the defendant's strategy so that he cross-examines only when he knows that the plaintiff will fail the test. As a result the court's beliefs are more favorable to the plaintiff when he submits $(x_1, \emptyset)$ and is not cross-examined as this now suggests that he would pass the test. But this is of no consequence because the trial outcome remains $d((x_1, \emptyset), \emptyset, \emptyset) = 1$ as in the equilibrium discussed above.

6 Concluding Remarks

Posner (1999, p. 1543) remarks that: "A principal social value of the right of cross-examination is deterrent: the threat of cross-examination deters some witnesses from testifying at all and others from giving false or misleading evidence. Merely observing cross-examination, therefore, does not give a complete picture of its social value."

[24] When the defendant plays a pure strategy, $\widetilde{e}_D = \hat{e}_D = e_D^{ac}$. The conditions in Lemma 5b then reduce to that in Lemma 5. When the strategy is mixed, it can be shown that e_D^{ac} and η_D^{ac} are negatively correlated, hence $\hat{e}_D < \widetilde{e}_D$.

[25] "Never, never, never, on cross-examination ask a witness a question you don't already know the answer to, was a tenet I absorbed with my baby-food. Do it, and you'll often get an answer you don't want, an answer that might wreck your case." (Harper Lee, *To Kill a Mockingbird*, 1960).

We studied a situation where the threat of cross-examination does indeed deter (i.e., reduce the probability of) potentially misleading reports. However, we find that cross-examination can improve adjudication only if does not deter too much. By itself, deterrence is detrimental to accuracy when the adjudicator is a sophisticated decision maker who anyway discounts reports with the appropriate skepticism. Moreover, the usefulness or significance of a report depend not only on its content and sender but also on how likely it is that the other party possesses countervailing evidence. The latter is less likely when cross-examination is allowed because of strategic interactions in the parties' incentives to acquire evidence. As a result, decisions need not be improved.

We discussed the merits of cross-examination solely with respect to accuracy in fact-finding. It may be that the decision maker (or society) is concerned both with accuracy and with the parties' costs (see again Posner 1999, among others). In our analysis, allowing cross-examination typically reduces the parties' expenditure. In particular, there will be situations where the right of cross-examination will both improve accuracy and reduce costs. Even when accuracy is not improved, society may nevertheless still be better off given how it trades-off accuracy and procedural costs.

Finally, it may be remarked that, in our simple set-up, only one party may find it useful to invest in cross-examination. This follows mechanically from our assumption that the potential evidence consists of at most two pieces of information. The party with the burden of proof has an incentive to submit incomplete and therefore potentially misleading evidence, which the other party may attempt to rebut through cross-examination or by disclosing additional evidence. In the latter case, that party's report may itself be misleading if the potential evidence consists of more than two pieces. Relaxing the assumption on the structure of evidence, our approach can therefore be generalized to allow for bilateral cross-examination.

Appendix

To shorten notation we write P_{ij} for $P(x_i, y_j)$ and $P_{ij|\omega_t}$ for $P(x_i, y_j \mid \omega_t)$, $i, j, t = 0, 1$.

Proof of Lemma 1. Let P and D deviate from their equilibrium strategies with the probabilities ε_P and ε_D respectively. When $m_P = (x_0, \emptyset)$ and $m_D = \emptyset$, then P deviated whether the complete evidence was (x_0, y_0) or (x_0, y_1) while D did not. Therefore

$$\Pr(m_P = (x_0, \emptyset), m_D = \emptyset) = (P_{00} + P_{01}) e_P \varepsilon_P (1 - e_D \varepsilon_D).$$

Applying Bayes' rule,

$$\begin{aligned}
\beta((x_0, \emptyset), \emptyset) &= \frac{p \Pr(m_P = (x_0, \emptyset), m_D = \emptyset \mid \omega_1)}{\Pr(m_P = (x_0, \emptyset), m_D = \emptyset)} \\
&= \frac{p(P_{00|\omega_1} + P_{01|\omega_1}) e_P \varepsilon_P (1 - e_D \varepsilon_D)}{(P_{00} + P_{01}) e_P \varepsilon_P (1 - e_D \varepsilon_D)} \\
&= \Pr(\omega_1 \mid x_0) < \frac{1}{2}.
\end{aligned}$$

For $m_P = \emptyset$ and $m_D = (x_0, \emptyset)$, D deviated and P also did if he had access to the evidence (x_0, y_1). Therefore

$$\begin{aligned}
\Pr(m_P = \emptyset, m_D = (x_0, \emptyset)) = {} & P_{00}[1 - e_P + e_P(1 - \varepsilon_P)] e_D \varepsilon_D \\
& + P_{01}[1 - e_P + e_P(\theta \varepsilon_P + (1 - \theta)(1 - \varepsilon_P))] e_D \varepsilon_D
\end{aligned}$$

From Bayes' rule and letting ε_P tend to zero,

$$\begin{aligned}
\beta(\emptyset, (x_0, \emptyset)) &= \frac{p[P_{00|\omega_1} + P_{01|\omega_1}(1 - \theta e_P)]}{P_{00} + P_{01}(1 - \theta e_P)} \\
&\leq \Pr(\omega_1 \mid x_0).
\end{aligned}$$

Similar arguments apply for the other cases. ■

Proof of Lemma 2. We complete the argument in the text by proving that $e_P^{ad} > e_D^{ad}$. Because $C'' > 0$, the claim is equivalent to $C'(e_P^{ad}) > C'(e_D^{ad})$ and therefore, using (7) and (8), to

$$P_{11} + \theta P_{01} + (1 - \theta e_D^{ad}) P_{10} > e_P^{ad} \theta P_{10}.$$

The left-hand side is decreasing in e_D^{ad} and the right-hand side increasing in e_P^{ad}. Therefore, it suffices that the inequality holds at $e_P^{ad} = e_D^{ad} = 1$, i.e.,

$$\psi(\theta) \equiv P_{11} + \theta P_{01} + (1 - 2\theta) P_{10} > 0. \tag{37}$$

Because $\psi(\theta)$ is linear in θ and $\psi(0) > 0$, (37) holds for all $\theta \in [0,1]$ if $\psi(1) > 0$. Now

$$\begin{aligned}\psi(1) &= P_{01} + (P_{11} + P_{10}) - 2P_{10} \\ &= P_{01} + [pq + (1-p)(1-q)] - 2[pq(1-h) + (1-p)(1-q)h] \\ &= P_{01} + (2h-1)[pq - (1-p)(1-q)] > 0, \qquad (38)\end{aligned}$$

where the inequality follows from Assumption 1. ■

Proof of Lemma 3. From Bayes' rule and given the communication strategies (1) and (2),

$$\beta((x_1, \emptyset), \emptyset) = \frac{p\{P_{11|\omega_1}(1-\theta) + P_{10|\omega_1}(1-\theta e_D)\}e_P}{\{P_{11}(1-\theta) + P_{10}(1-\theta e_D)\}e_P} \qquad (39)$$

Hence $\beta((x_1, \emptyset), \emptyset) > \frac{1}{2}$ is equivalent to

$$\begin{aligned}&p\{P_{11|\omega_1}(1-\theta) + P_{10|\omega_1}(1-\theta e_D)\} \\ &\quad > (1-p)\{P_{11|\omega_0}(1-\theta) + P_{10|\omega_0}(1-\theta e_D)\}. \qquad (40)\end{aligned}$$

Substituting for the conditional probabilities yields

$$p[qh(1-\theta)+q(1-h)(1-\theta e_D)] > (1-p)[(1-q)(1-h)(1-\theta)+(1-q)h(1-\theta e_D)$$

or equivalently

$$[pqh - (1-p)(1-q)(1-h)](1-\theta) > [(1-p)(1-q)h - pq(1-h)](1-\theta e_D).$$

This is condition (9) with k_P and k_D as defined in the lemma, where $k_P > k_D$ follows from Assumption 1. The condition (9) is satisfied for all $e_D \leq 1$ if $(1-\theta)k_P > k_D$, equivalently $\theta < \theta_a$ as defined. The rest of the proof follows trivially. ■

Proof of Proposition 1. We first show that $\beta(\emptyset, \emptyset) < \frac{1}{2}$ under both the Passive and Active Defendant strategy profiles. Applying Bayes' rule,

$$\beta(\emptyset, \emptyset) = \frac{p\,\Pr(m_P = \emptyset, m_D = \emptyset \mid \omega_1)}{\Pr(m_P = \emptyset, m_D = \emptyset)}$$

so that $\beta(\emptyset, \emptyset) < \frac{1}{2}$ if

$$\frac{\Pr(m_P = \emptyset, m_D = \emptyset \mid \omega_0)}{\Pr(m_P = \emptyset, m_D = \emptyset \mid \omega_1)} > \frac{p}{1-p}. \tag{41}$$

Because $p \leq \frac{1}{2}$, the inequality (41) holds if the left-hand side is greater than unity, equivalently if

$$\xi \equiv \Pr(m_P = \emptyset, m_D = \emptyset \mid \omega_0) - \Pr(m_P = \emptyset, m_D = \emptyset \mid \omega_1) > 0.$$

Given the communication strategies,

$$\begin{aligned}\Pr(m_P = \emptyset, m_D = \emptyset \mid \omega_i) = P_{11|\omega_i}(1-e_P) + P_{10|\omega_i}(1-e_P)(1-\theta e_D) \\ + P_{01|\omega_i}(1-\theta e_P) + P_{00|\omega_i}.\end{aligned}$$

Hence

$$\begin{aligned}\xi = (P_{11|\omega_0} - P_{11|\omega_1})(1-e_P) + (P_{10|\omega_0} - P_{10|\omega_1})(1-e_P)(1-\theta e_D) \\ (P_{00|\omega_0} - P_{00|\omega_1}) + (P_{01|\omega_0} - P_{01|\omega_1})(1-\theta e_P).\end{aligned}$$

Substituting for the conditional probabilities and rearranging,

$$\begin{aligned}\xi &= ((1-q)(1-h) - qh)(1-e_P) + ((1-q)h - q(1-h))(1-e_P)(1-\theta e_D) \\ &\quad +(q(1-h) - (1-q)h)(1-\theta e_P) + (qh - (1-q)(1-h)) \\ &= (2q-1)e_P + \theta(h-q)(e_P - e_D + e_P e_D).\end{aligned}$$

It follows that $\xi > 0$ if $e_P > e_D \geq 0$, which is the case in either the *pd* or the *ad*-equilibrium as shown in Lemma 2. We show next that at least one of these equilibria exists.

Case $\theta < \theta_a$. By Lemma 3, $\beta((x_1, \emptyset), \emptyset) > \frac{1}{2}$ and therefore $d((x_1, \emptyset), \emptyset) = 1$ irrespective of e_D. Hence the plaintiff strictly gains by submitting the evidence $(x_1, \emptyset)$. From Lemma 2, the unique equilibrium is then the *ad*-equilibrium with $e_P > e_D > 0$ solving (7) and (8).

Case $\theta \geq \theta_a$. By Lemma 3, $\beta((x_1, \emptyset), \emptyset) \leq \frac{1}{2}$ and therefore $d((x_1, \emptyset), \emptyset) = 0$ if $e_D = 0$. Therefore the *pd*-equilibrium with $e_P > e_D = 0$ then exists. If the *ad*-investment game has a solution $e_D > \varphi(\theta)$, then an *ad*-equilibrium also exists. We conclude by showing that this cannot arise for θ sufficiently large. Because $C'(1) \geq 1$, the conditions (7) and (8) imply $e_D < \bar{e} < 1$ where $\bar{e}$ solves $C'(e) = e_P P_{10}\theta$. Because $\varphi(\theta)$ is strictly increasing and $\varphi(1) = 1$, it follows that $e_D^{ad}(\theta) < \bar{e} \leq \varphi(\theta)$ for all $\theta \geq \varphi^{-1}(\bar{e})$. ■

Proof of Proposition 2. From the first-order conditions (7) and (6),

$$\begin{aligned} C'(e_P^{pd}) &= \theta(P_{11} + P_{01}) \\ &< \theta(P_{11} + P_{01}) + (1-\theta)(P_{11} + P_{10}) \\ &= P_{11} + \theta P_{01} + P_{10}(1-\theta) \\ &< P_{11} + \theta P_{01} + P_{10}(1 - e_D^{ad}\theta) \\ &= C'(e_P^{ad}). \end{aligned}$$

That $e_P^{ad} > e_P^{pd}$ then follows from $C'' > 0$.

In the pd-equilibirum, $\Pr(d = 1 \mid \omega_1) = e_P^{pd}\theta h$ and $\Pr(d = 1 \mid \omega_0) = e_P^{pd}\theta(1-h)$. Substituting in (10) then yields (11) with Δ^{pd} as defined in the text. In the ad-equilibrium,

$$\begin{aligned} \Pr\Big(d = 1 \mid \omega_1\Big) &= e_P^{ad}[qh + q(1-h)(1 - e_D^{ad}\theta) + \theta(1-q)h], \\ \Pr\Big(d = 1 \mid \omega_0\Big) &= e_P^{ad}[(1-q)(1-h) + (1-q)h(1 - e_D^{ad}\theta) + \theta q(1-h)]. \end{aligned}$$

Substituting in (10) yields (13) with

$$\begin{aligned} \Delta^{ad} &= \theta\,[p(1-q)h - (1-p)q(1-h)] + [pqh - (1-p)(1-q)(1-h)] \\ &\quad - [(1-p)(1-q)h - pq(1-h)]\,(1 - e_D^{ad}\theta) \\ &= \theta\,[ph - (1-p)(1-h)] + [pqh - (1-p)(1-q)(1-h)]\,(1-\theta) \\ &\quad - [(1-p)(1-q)h - pq(1-h)]\,(1 - e_D^{ad}\theta) \\ &= \Delta^{pd} + \Big[k_P(1-\theta) - k_D(1 - e_D^{ad}\theta)\Big].■ \end{aligned}$$

Proof of Lemma 4 and 4b. We first prove the more general formulation 4b without imposing Assumption 2. Let e_P^{ac} and $\sigma(e_D, \eta_D)$ be a solution to the AC investment game. Then

$$\begin{aligned} C'(e_P^{ac}) &= P_{01}\theta + P_{11}E_\sigma[1 - \eta_D(1-\theta)(1-v)] \\ &\quad + P_{10}E_\sigma\{(1 - e_D\theta)[1 - \eta_D(\theta v + (1-\theta)(1-v))]\} \end{aligned} \tag{42}$$

and for any (e_D^{ac}, η_D^{ac}) in the support of σ,

$$C'(e_D^{ac}) = e_P^{ac} P_{10}\theta[1 - \eta_D^{ac}(\theta v + (1-\theta)(1-v))], \tag{43}$$

$$K'(\eta_D^{ac}) = e_P^{ac}\{P_{11}(1-\theta)(1-v)] + P_{10}(1-e_D^{ac}\theta)(\theta v + (1-\theta)(1-v))]\}. \tag{44}$$

The right-hand side of (42) is positive so that $e_P^{ac} > 0$, implying e_D^{ac}, $\eta_D^{ac} > 0$.

Next we show uniqueness with respect to e_P^{ac}. In (18) let $\overline{s} = S(0,0)$ and $\underline{s} = S(1,1)$. The defendant's best response can be decomposed as first minimizing the cost of $s \in (\underline{s}, \overline{s}]$ and then choosing s. The first step yields

$$G(s) = \min_{e_D, \eta_D \geq 0} C(e_D) + K(\eta_D) \text{ s.t. } S(e_D, \eta_D) \leq s. \tag{45}$$

$G(s)$ is continuous, strictly decreasing and with $G(\overline{s}) = G'(\overline{s}) = 0$. In the second step, the defendant maximizes $\pi_D = -e_P s - G(s)$. Let $\Sigma(e_P)$ denote the solution set. Because $\partial \pi_D / \partial s$ is decreasing in e_P, from Edlin and Shannon's (1998) strict monotonicity theorem, $e_P'' > e_P'$ implies $s'' < s'$ for all $s' \in \Sigma(e_P')$ and $s'' \in \Sigma(e_P'')$.

Now let the probability distribution $\nu(s)$ be a strategy chosen by the defendant. The plaintiff's best response solves $C'(e_P) = E_\nu(s)$, hence e_P is increasing in $E_\nu(s)$. Finally let e_P^* and $\nu^*(s)$ be a solution to the *ac*-investment game. Then $C'(e_P^*) = E_{\nu^*}(s)$ and the support of ν^* is contained in $\Sigma(e_P^*)$. If another solution existed, say e_P^{**} and ν^{**} where $e_P^{**} > e_P^*$, then the support of ν^{**} must be contained in $\Sigma(e_P^{**})$. By the argument above, this implies $E_{\nu^{**}}(s) < E_{\nu^*}(s)$, which is inconsistent with $e_P^{**} > e_P^*$.

It remains to proove that $e_P^{ac} > E_\sigma(e_D) \equiv \overline{e}_D^{ac}$. The inequality is equivalent to

$$C'(e_P^{ac}) > C'(\overline{e}_D^{ac}). \tag{46}$$

We first show that

$$C'(e_P^{ac}) > E_\sigma \left[C'(e_D)\right]. \tag{47}$$

Inequality (46) then follows from Jensen's inequality and our assumption that $C''' \geq 0$, implying $E_\sigma\left[C'(e_D)\right] \geq C'(\overline{e}_D^{ac})$. Using (42) and (43), inequality (47) is equivalent to

$$\begin{aligned}
&P_{01}\theta + P_{11}[1 - \overline{\eta}_D^{ac}(1-\theta)(1-v)] \\
&+ P_{10}E_\sigma\{(1 - e_D\theta)[1 - \eta_D(\theta v + (1-\theta)(1-v))]\} \\
&- e_P^{ac}P_{10}\theta[1 - \overline{\eta}_D^{ac}(\theta v + (1-\theta)(1-v))] > 0
\end{aligned}$$

where $\overline{\eta}_D \equiv E_\sigma(\eta_D)$. A sufficient condition for the latter inequality (i.e., set $e_D \equiv e_P^{ac} = 1$) is

$$\begin{aligned} Q(\theta) &\equiv P_{01}\theta + P_{11}[1 - \overline{\eta}_D^{ac}(1-\theta)(1-v)] \\ &\quad + P_{10}\{(1-\theta)[1 - \overline{\eta}_D^{ac}(\theta v + (1-\theta)(1-v))]\} \\ &\quad - P_{10}\theta[1 - \overline{\eta}_D^{ac}(\theta v + (1-\theta)(1-v))] \\ &= P_{01}\theta + P_{11}[1 - \overline{\eta}_D^{ac}(1-\theta)(1-v)] \\ &\quad + P_{10}[1 - \overline{\eta}_D^{ac}(\theta v + (1-\theta)(1-v))](1-2\theta) \\ &> 0 \text{ for all } \theta \in [0,1]. \end{aligned}$$

For $\theta < \frac{1}{2}$, $Q(\theta) > 0$. For $\theta \geq \frac{1}{2}$,

$$Q(\theta) > P_{01}\theta + P_{11}[1 - \overline{\eta}_D^{ac}(1-\theta)(1-v)] + P_{10}(1-2\theta) \equiv T(\theta)$$

where $T(\theta)$ is linear in θ with $T(\frac{1}{2}) > 0$ and

$$T(1) = P_{01} + P_{11} - P_{10}(1 - \overline{\eta}_D v) > \psi(1) > 0,$$

where the function ψ is as defined in the proof of Lemma 2; see inequality (38). Thus inequality (46) holds.

We complete the proof by showing that, under Assumption 2, (e_D^{ac}, η_D^{ac}) is unique, i.e., the defendant plays a pure strategy. The defendant maximize

$$\pi(e_D, \eta_D) = -e_P^{ac} S(e_D, \eta_D) - C(e_D) - K(\eta_D)$$

Necessary conditions are (rewriting (43) and (44)):

$$\begin{aligned} -e_P^{ac} S_{e_D}(e_D, \eta_D) - C'(e_D) &= 0, \qquad (48) \\ -e_P^{ac} S_{\eta_D}(e_D, \eta_D) - K'(\eta_D) &= 0. \qquad (49) \end{aligned}$$

We show that Assumption 2 implies that, for all (e_D, η_D) satisfying (48) and (49),

$$C''(e_D)K''(\eta_D) - (e_P^{ac} S_{e_D\eta_D}(e_D, \eta_D))^2 > 0. \qquad (50)$$

Any stationary point is therefore a strict local maximum, hence a local maximum is the unique global maximum. Condition (50) is equivalent to: for all (e_D, η_D) satisfying (48) and (49),

$$\frac{C''(e_D)}{C'(e_D)} \frac{K''(\eta_D)}{K'(\eta_D)} > \frac{(S_{e_D\eta_D}(e_D, \eta_D))^2}{S_{e_D}(e_D, \eta_D) S_{\eta_D}(e_D, \eta_D)}. \qquad (51)$$

Writing $\overline{\theta} \equiv \theta v + (1-\theta)(1-v)$, for all (e_D, η_D),

$$\begin{aligned}
\frac{S^2_{e_D\eta_D}}{S_{e_D}S_{\eta_D}} &= \frac{\left(\theta\overline{\theta}P_{10}\right)^2}{\left[\theta P_{10}(1-\eta_D\overline{\theta}\right]\left[P_{11}(1-\theta)(1-v)+P_{10}(1-e_D\theta)\overline{\theta}\right]} \\
&< \frac{\left(\theta\overline{\theta}P_{10}\right)^2}{\left[\theta P_{10}(1-\eta_D\overline{\theta}\right]\left[P_{10}(1-e_D\theta)\overline{\theta}\right]} \\
&= \frac{\theta\overline{\theta}}{(1-\eta_D\overline{\theta})(1-e_D\theta)} \\
&< \frac{\theta\overline{\theta}}{(1-\overline{\theta})(1-\theta)}
\end{aligned}$$

Assumption 2 is then easily seen to imply (51).■

Proof of Lemma 5 and 5b. We prove only 5b, first showing (33). From Bayes' rule and given the communication and cross-examination strategies,

$$\beta((x_1,\emptyset),\emptyset,\emptyset) = \frac{pE_\sigma\{[P_{11|\omega_1}(1-\theta)+P_{10|\omega_1}(1-\theta e_D)](1-\eta_D)\}e_P}{E_\sigma\{[P_{11}(1-\theta)+P_{10}(1-\theta e_D)(1-\eta_D)]\}e_P}.$$

Hence $\beta((x_1,\emptyset),\emptyset,\emptyset) > \frac{1}{2}$ if

$$\begin{aligned}
& pE_\sigma\{[P_{11|\omega_1}(1-\theta)+P_{10|\omega_1}(1-\theta e_D)](1-\eta_D)\} \\
> \; & (1-p)E_\sigma\{[P_{11|\omega_0}(1-\theta)+P_{10|\omega_0}(1-\theta e_D)](1-\eta_D)\}.
\end{aligned}$$

Using (34) and dividing by $E_\sigma(1-\eta_D)$, the above is equivalent to

$$\begin{aligned}
& p\{P_{11|\omega_1}(1-\theta)+P_{10|\omega_1}(1-\theta\widetilde{e}_D)\} \\
& > (1-p)\{P_{11|\omega_0}(1-\theta)+P_{10|\omega_0}(1-\theta\widetilde{e}_D)\}.
\end{aligned}$$

This has the same form as (40) in the proof of Lemma 3. So a similar argument proves (33).

To prove the second inequality in (35),

$$\beta((x_1,\emptyset),\emptyset,1) = \frac{pE_\sigma\{[P_{11|\omega_1}(1-\theta)(1-v)+P_{10|\omega_1}(1-\theta e_D)(\theta v+(1-\theta)(1-v))]\eta_D\}e_P}{E_\sigma\{[P_{11}(1-\theta)(1-v)+P_{10}(1-\theta e_D)(\theta v+(1-\theta)(1-v))]\eta_D]\}e_P}.$$

Therefore $\beta((x_1,\emptyset),\emptyset,1) \leq \frac{1}{2}$ if

$$pE_\sigma\{[P_{11|\omega_1}(1-\theta)(1-v)+P_{10|\omega_1}(1-\theta e_D)(\theta v+(1-\theta)(1-v))]\eta_D\}$$
$$\leq (1-p)E_\sigma\{[P_{11|\omega_0}(1-\theta)(1-v)+P_{10|\omega_0}(1-\theta e_D)(\theta v+(1-\theta)(1-v))]\eta_D\}.$$

Dividing by $\theta v+(1-\theta)(1-v)$ and using (15), the above is equivalent to

$$pE_\sigma\{[P_{11|\omega_1}(1-\hat{\theta}_1)+P_{10|\omega_1}(1-\theta e_D)]\eta_D\}$$
$$\leq (1-p)E_\sigma\{[P_{11|\omega_0}(1-\hat{\theta}_1)+P_{10|\omega_0}(1-\theta e_D)]\eta_D\}.$$

Using (36) and dividing by $E_\sigma(\eta_D)$, this in turn is equivalent to

$$p\{P_{11|\omega_1}(1-\hat{\theta}_1)+P_{10|\omega_1}(1-\theta\hat{e}_D)\}$$
$$\leq (1-p)\{P_{11|\omega_0}(1-\hat{\theta}_1)+P_{10|\omega_0}(1-\theta\hat{e}_D)\}.$$

Again the expression is as in (40) but with the reverse inequality. The argument for the first inequality in (35) is similar. ■

Proof of Proposition 3 and 3b. We show that $E_\sigma(e_D) < e_D^{ad}$. The inequality follows trivially if $e_D^{ac} < e_D^{ad}$ for all e_D^{ac} in the support of σ. So suppose there exists e_D^{ac} in the support such that $e_D^{ac} \geq e_D^{ad}$. Comparing (8) and (43) and recalling that $\eta_D^{ac} > 0$ in the support, we must then have $e_P^{ac} > e_P^{ad}$ or equivalently $C'(e_P^{ac}) > C'(e_P^{ad})$. From (7) and (42), the latter inequality is equivalent to

$$P_{11}E_\sigma[1-\eta_D(1-\theta)(1-v)]+P_{10}E_\sigma\{(1-e_D\theta)[1-\eta_D(\theta v+(1-\theta)(1-v))]\}$$
$$> P_{11}+P_{10}(1-e_D^{ad}\theta).$$

A necessary condition is

$$P_{11}+P_{10}E_\sigma(1-e_D\theta) > P_{11}+P_{10}(1-e_D^{ad}\theta),$$

which is equivalent to $E_\sigma(e_D) < e_D^{ad}$. ■

Proof of Proposition 4. For the *pd*-outcome, the claim follows directly from Proposition 2. In the *ac*-outcome, and without imposing Assumption 2, the probability of correct adjudication satisfies (10) with

$$\begin{aligned}\Pr\left(d=1 \mid \omega_1\right) &= e_P^{ac}\{P_{11|\omega_1}[1-(1-\theta)(1-v)E_\sigma(\eta_D)] \\ &\quad + P_{10|\omega_1}E_\sigma[(1-\theta e_D)(1-\overline{\theta}\eta_D)] + P_{01|\omega_1}\theta\}\end{aligned}$$

where $\overline{\theta} \equiv \theta v + (1-\theta)(1-v)$. Substituting in (10) and writing the probabilities explicitly yields

$$\begin{aligned}
\Delta^{ac} &= [pqh - (1-p)(1-q)(1-h)][1-(1-\theta)(1-v)E_\sigma(\eta_D))] \\
&\quad - [(1-p)(1-q)h - pq(1-h)]E_\sigma[(1-\theta e_D)(1-\overline{\theta}\eta_D)] \\
&\quad + [p(1-q)h - (1-p)q(1-h)]\theta.
\end{aligned}$$

Rearranging and using k_P and k_D as defined in Lemma 3,

$$\Delta^{ac} = \Delta^{pd} + k_P(1-\theta)[1-(1-v)E_\sigma(\eta_D)] - k_D E_\sigma[(1-\theta e_D)(1-\overline{\theta}\eta_D)]. \quad (52)$$

Comparing with (14), $\Delta^{ac} \geq \Delta^{ad}$ if and only if

$$\frac{k_P(1-\theta)(1-v)E_\sigma(\eta_D)}{k_D} + E_\sigma[(1-\theta e_D)(1-\overline{\theta}\eta_D)] \leq 1 - \theta e_D^{ad}. \quad (53)$$

From (7) and (42) and the convexity of the investigation cost, $e_P^{ac} \geq e_P^{ad}$ if and only if

$$-\frac{P_{11}(1-\theta)(1-v)E_\sigma(\eta_D)}{P_{10}} + E_\sigma[(1-\theta e_D)(1-\overline{\theta}\eta_D)] \geq 1 - \theta e_D^{ad}. \quad (54)$$

Clearly (53) and (54) cannot simultaneously hold, implying that $\Delta^{ac} \geq \Delta^{ad}$ together with $e_P^{ac} \geq e_P^{ad}$ is impossible, which is equivalent to "$\Delta^{ac} < \Delta^{ad}$ or $e_P^{ac} < e_P^{ad}$".

We complete the argument by showing that (25) is equivalent to (26). From (52) and writing $E_\sigma(\eta_D) = \overline{\eta}_D^{ac}$,

$$\begin{aligned}
\Delta^{ac} &= \Delta^{pd} + k_P[(1-\overline{\eta}_D^{ac})(1-\theta) + \overline{\eta}_D^{ac}(1-\theta)v] \\
&\quad - k_D E_\sigma[(1-\eta_D)(1-\theta e_D) + \eta_D(1-\overline{\theta})(1-\theta e_D)] \\
&= \Delta^{pd} + (1-\overline{\eta}_D^{ac})\left\{k_P(1-\theta) - k_D\left(1-\theta\frac{E_\sigma[e_D(1-\eta_D)]}{1-\overline{\eta}_D}\right)\right\} \\
&\quad + \overline{\eta}_D^{ac}(1-\overline{\theta})\left\{k_P\frac{(1-\theta)v}{1-\overline{\theta}} - k_D\left(1-\theta\frac{E_\sigma(e_D\eta_D)}{\overline{\eta}_D}\right)\right\} \\
&= \Delta^{pd} + (1-\overline{\eta}_D^{ac})\,[k_P(1-\theta) - k_D(1-\theta\widetilde{e}_D^{ac})] \\
&\quad + \overline{\eta}_D^{ac}(1-\overline{\theta})[k_P(1-\hat{\theta}_0) - k_D(1-\theta\hat{e}_D^{ac})]
\end{aligned}$$

where $\widetilde{e}_D^{ac}$ and $\hat{e}_D^{ac}$ are as defined in (34) and (36). When the defendant plays a pure strategy at equilibrium, $\widetilde{e}_D^{ac} = \hat{e}_D^{ac} = e_D^{ac}$ and the above yields (26). ■

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