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Bayesian Estimation of Financial Frictions: An Encompassing View

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Abstract

This paper compares the environments in Bernanke et al. (1999) and Gertler and Karadi (2011), two popular frameworks used to incorporate financial frictions in macroeconomic modelling. We show that the key practical difference between the two frameworks lies in their implications for the link between leverage and expected future spreads of capital returns over safe rates: while the former pairs leverage to one-period-hence such spreads, the latter connects it to a distributed lag of all future spreads. We argue that this difference between the two frameworks is more crucial than the distinction often discussed in the literature, which is related to the specific location of the friction on the borrower-intermediary-entrepreneur financing chain. The paper then compares quantitative versions of the frameworks, estimated using Bayesian procedures and decoupling parameter settings related to steady states from those involving the economy's dynamic solution around that steady state. We find that when this flexible approach in used, the friction proposed by Gertler and Karadi (2011), which emphasize long-term forward-looking behavior in the leverage equation, is preferred by aggregate data.

Key words: Financial frictions, DSGE Models, Bayesian estimation.

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1 Introduction

Macroeconomists long considered financial markets to be a veil which, although crucial for channelling funds from savers to borrowers, played a negligible role in originating and propagating business cycle-type fluctuations. Work by Bernanke (1983), among others, contributed to change this vision by highlighting the role played by a credit channel linking events in financial markets to real sector outcomes during the Great Depression. An extensive empirical literature has since confirmed that the financial health of borrowers, or of the financial intermediaries lending to them, has important implications for macroeconomic outcomes.[1]

Modeling frameworks have been proposed to operationalize such real-financial linkages and embed them within macroeconomic models. One popular such framework originates from work by Carlstrom and Fuerst (1997) and Bernanke et al. (1999): it assumes that a costly-state-verification (CSV) problem affects the relationship between borrowing firms and lenders, in that realized returns of firm projects are observable by lenders only after paying a monitoring cost. This leaves borrowing firms with an incentive to underreport results in order to disengage from their obligations. In response, lenders require that borrowers contribute their own net worth to the financing of projects. The evolution of net worth thus becomes an important variable and governs how much a firm can borrow; at the macroeconomic scale, this implies that lending, investment and the overall pace of economic activity depend on the evolution of aggregate net worth.

Alternatively, Gertler and Karadi (2011) assume the presence of a costly enforcement problem wherein borrowers can divert a fraction of the borrowed funds from the underlying project in a manner unrecoverable by the lender, whose only recourse is to force the borrower into default and thus ban the borrower from credit markets. The upshot of this environment is that lenders ration borrowers up to the point where they find it preferable to pay back loans rather than default and forfeit the long-term value of access to financial markets. The Bernanke et al. (1999) and Gertler and Karadi (2011) frameworks thus appear at first quite distinct, relying on conceptually different information or enforcement restrictions. In addition, these authors locate their friction at different junctions of the depositor-lender-borrower financing chain: while Bernanke et al. (1999) assume it is the lender-borrower link that is affected by the CSV friction, Gertler and Karadi (2011), by contrast, assume that the depositor-lender connection is where the costly enforcement problem occurs.

This paper shows that the key practical distinction between the two frameworks however lies in the dynamic relationship between leverage and future project returns they imply. Indeed, while the Bernanke et al. (1999) friction entails a well-known relation between current leverage and the one-period-ahead spread of capital returns over the risk-free rate, we show that the one from Gertler and Karadi (2011) links leverage to a distributed sum of all such future spreads. As such, this paper argues that discussions about the specific actors affected by the financial friction of each framework, which dominate the literature, may be of secondary importance relative to the dynamic implications of the modeled environment. Instead, these frictions may instead be interpreted as applying to the broad link between savings (the household side) and the uses of savings (ie. investment in and management of physical capital) by a combined intermediary/entrepreneurial block.[2]

[1] Important contributions include Gertler and Gilchrist (1994) and Gilchrist and Himmelberg (1995), who show that cash flows and other financial results of firms influence their access to financial markets beyond the influence of fundamentals. It also includes work by Peek and Rosengren (1997), Kishan and Opiela (2000) or Kashyap and Stein (2000) showing that the financial health of banks and other intermediaries importantly affects their lending ability.

[2] Research on financial frictions most often locates agency problems on the link between borrowing firms and lending intermediaries while assuming that the relationship between lending intermediaries and their own sources of funds is frictionless (Bernanke et al., 1999; Meier and Muller, 2006; Christensen and Dib, 2008; De Graeve, 2008; Queijo von Heideken, 2009). A smaller part of the literature instead posits that agency problems affect the link between depositors

To test the implications of our argument, we use Bayesian methods to estimates three versions of the medium-scale Smets and Wouters (2003, 2007) macromodel: a first version incorporating no financial frictions and thus serving as a benchmark and two other versions respectively embedding the Bernanke et al. (1999) and Gertler and Karadi (2011) frictions. Our Bayesian estimation is flexible, in the sense that the parameters in the leverage equation, which are often calibrated as a byproduct of solving the economy's steady state, are instead included in the Bayesian procedure (with those calibrated values used as priors). As such, our procedure aims to usefully decouple the process of solving a financial-friction model's steady-state from that of establishing its dynamic adjustment around that steady state, a strategy that may be particularly relevant for applied work with medium-scale macromodels.[3]

Our main findings are as follows. First, models with financial frictions are robustly preferred by the data to the benchmark that incorporates no such friction: this confirms results obtained by a large literature assessing the implications of financial frictions (Meier and Muller, 2006; Christensen and Dib, 2008; Queijo von Heideken, 2009; De Graeve, 2008; Brzoza-Brzezina and Kolasa, 2013). Second, the friction proposed by Gertler and Karadi (2011), in which leverage determinants are more forward looking than in Bernanke et al. (1999), is overall preferred by aggregate data but only when our flexible approach decoupling steady state and dynamic computations is employed. Third, departing from calibrated or steady-state-linked values for the parameters in the leverage equation brings some new insights into the working of the models' financial frictions: for instance, the posterior mean of the Bernanke et al. (1999) leverage parameter is consistently smaller than its prior mean, while the corresponding posterior means for the Gertler and Karadi (2011) parameters are also substantially different from their priors.

A closely related contribution to our work is represented by Villa (2013) and Villa (2016), which also assess performance for the Bernanke et al. (1999) versus Gertler and Karadi (2011) environments using Bayesian methods. We depart from that work in two important ways. First, Villa (2013, 2016) follows the standard arguments in the literature and assumes the CSV friction appears on the lending market's demand-side (entrepreneurs), while the costly enforcement one affects that market's supply-side (financial intermediaries). A finding that the Gertler and Karadi (2011) environment is preferred by the data is thus interpreted as showing frictions on the bank side are more prevalent in real economies. As noted above, we argue that the specific location of the friction on the depositor-lender-borrower axis is secondary relative to the dynamic structure that the framework implies for leverage. Second, Villa (2013, 2016) uses steady-state-derived values for the parameters linked to the frictions and, as a result, does not include them in the Bayesian procedure: by contrast, we demonstrate the benefit of decoupling the tasks of solving for the steady state and solving for the dynamic solution. Overall, our paper suggests that frictions which at fist appear conceptually very different may be usefully interpreted in an encompassing manner by focusing on their implications for the dynamic link between savings and uses-of-savings blocks.

The rest of this paper is structured as follows. Section 2 below presents the benchmark model

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and financial intermediaries while keeping the the lender-entrepreneur leg exempt from frictions. (Parlour and Plantin, 2008; Gertler and Karadi, 2011; Plantin, 2015). Some environments incorporate agency problems on both of these links (Holmstrom and Tirole, 1997; Meh and Moran, 2010). This paper's argument is that for the purpose of applied work fitting aggregate date, this distinction may be secondary relative to the dynamic structure implied by the chosen framework.

[3]For example, solving for the steady state of the Gertler and Karadi (2011) environment requires the calibration of the parameter governing the extent of projet value borrowers can abscond with, or the effective discount factor of lenders. Since this calibration also affects the economy's dynamic adjustment around the steady state through a first-order approximate solution, setting the model's steady state also sets the economy's dynamic solution. Our argument is that the two operations can be usefully decoupled using a Bayesian procedure.

with no financial friction. Section 3 then describes how this benchmark is modified to include the two financial frictions. Since important building blocks of the three models are common, Section 3 focuses on the aspects that are modified by the presence of the financial frictions. Section 3 also shows that the key practical difference between these two financial-friction versions of the model lies in the implied dynamics for the relationship between current leverage and expected future returns to capital. Section 4 describes the Bayesian estimation approach and data that are used to estimate parameters and confront the models to aggregate data. Finally, Section 5 reports estimation results and our analysis of these results, while Section 6 concludes.

2 A model with no financial frictions

This section presents a New Keynesian model where financial frictions are absent. This model is based on the work of Smets and Wouters (2003, 2007) and will be used as a benchmark in our quantitative assessment. Its economy is populated by nine categories of agents: households, labour unions, labour packers, intermediate-good producers, retailers, final-good producers, capital producers, the monetary authority and the government.

Households supply labour services, consume and save, with our benchmark specification also assuming that they own the physical capital and decide how intensely to use it. This tight link between decisions about saving and those about capital accumulation and its management is relaxed in Section 3, when financial frictions are incorporated into the model and new agents –entrepreneurs– are introduced.

The labour market structure is one commonly adopted in New Keynesian-type models and is meant to facilitate the introduction of rigidities in the evolution of nominal wages. To this end, assume that *labour unions* differentiate households' homogenous labour services and resell them to *labour packers*, operating in a monopolistically competitive market structure that includes rigidities in wage-setting. The role of the *labour packers* is then to re-aggregate these labour –or union– types into a composite labour service sold to intermediate-goods producers. Under this representation of the labour market, consumption and hours worked are identical across households and the heterogeneity in quantities demanded for each labour type that result from wage-setting rigidities applies to the union.[4]

The structure of the market for goods is similar. As such, *retailers* purchase homogenous intermediate goods, differentiate them and resell each variety to final-goods producers within a monopolistic competition market structure that once again includes rigidities in price-setting. Final-good producers, like the labour packers above, aggregate these differentiated goods into a composite final good, operating in a competitive environment. Finally, intermediate-good producers use capital and labour services to produce the goods used as input by the retailers.

The model also includes capital producers that combine non-depreciated capital and final goods to create new capital goods sold to households, a monetary authority setting the nominal interest rate through a Taylor-type rule and a fiscal policy financing an exogenous stream of public expenditures via lump-sum taxes imposed on households. The dynamics of the model are governed six exogenous disturbances affecting general technical progress, investment-specific technology, monetary policy, government expenditures and, finally, mark-ups in price and wage-setting.

[4]This follows Schmitt-Grohé and Uribe (2006b). By contrast, Erceg et al. (2000) assume that heterogenous labour services are sold by households within monopolistically competitive markets so that any resulting heterogeneity in the demand for a specific labour type translates to hours worked by specific individuals. See Schmitt-Grohé and Uribe (2006a) for a discussion of these two alternative specifications for the labour market.

2.1 Households

A continuum of infinitely-lived households is present in the economy. The representative household's preferences are described by the utility function

$$U_t = ln(c_t - hC_{t-1}) - \frac{l_t^{1+\phi}}{1+\phi} \tag{1}$$

where $h \in (0,1)$ et $\phi > 0$ measure the degree of external habit in consumption and the inverse of the Frisch elasticity of labour supply, respectively.

At the start of period t, the representative household owns the quantity of physical capital k_t as well as bonds b_t. Income received during the period includes $R_t^H u_t k_t$ in capital income, where u_t is the utilisation rate of capital and R_t^H the rental rate for capital services.[5] Additional sources of income include arise from labour $\frac{W_t^h}{P_t} l_t$, where l_t represents hours worked and $\frac{W_t^h}{P_t}$ is the real wage, $q_t(1-\delta)k_t$, which results from selling the non depreciated capital at the end of the period (q_t is the price of one capital unit and δ is the depreciation rate), a transfer T_t from the government, a dividend Π_t from the ownership of firms and the financial return $R_{t-1}b_t$ from bond holdings. Such income must be sufficient to cover consumption expenditures c_t, the purchase of new bonds b_{t+1} and investment in new capital goods $q_t k_{t+1}$. The following budget constraint therefore applies:

$$c_t + b_{t+1} + q_t k_{t+1} \leq \frac{W_t^h}{P_t} l_t + R_{t-1} b_t + R_t^H u_t k_t - \upsilon(u_t) k_t + q_t(1-\delta)k_t + \Pi_t + T_t, \tag{2}$$

where the convex function $\upsilon(u_t)$ measures costs linked to the chosen utilisation rate of capital u_t.

The representative household's optimization problem is to choose values of c_t, b_{t+1}, l_t, k_{t+1} and u_t that maximise lifetime utility under the constraint of the budget constraint:

$$\max_{c_t, b_{t+1}, l_t, k_{t+1}, u_t} E_t \sum_{j=0}^{\infty} \beta^j U_{t+j}, \tag{3}$$

with respect to (1) and (2) and where β represents the discount factor. The necessary first-order conditions are as follows:

$$(c_t - hC_{t-1})^{-1} = \lambda_t; \tag{4}$$

$$\beta R_t E_t(\lambda_{t+1}) = \lambda_t; \tag{5}$$

$$l_t^{\phi} = \lambda_t \frac{W_t^h}{P_t}; \tag{6}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left[R_{t+1}^H u_{t+1} - \upsilon(u_{t+1}) + (1-\delta) q_{t+1} \right]; \tag{7}$$

$$R_t^H = \upsilon'(u_t); \tag{8}$$

with λ_t the Lagrange multiplier associated with the budget constraint.

For the purpose of interpreting the first-order conditions related to savings and investment, let r_t^k denote the gross return on savings allocated to capital goods in the preceding period so that

$$r_t^k = \frac{R_t^H u_t - \upsilon(u_t) + (1-\delta) q_t}{q_{t-1}}. \tag{9}$$

[5] A variable utilisation rate for physical capital is often used in this literature to break the tight relation between the capital stock and its rental rate (Christiano et al., 2005; Queijo von Heideken, 2009).

Using this definition and combining (5) and (7), one can show that up to a first-order approximation,

$$E_t\left(r_{t+1}^k\right) = R_t, \tag{10}$$

ie. the expected future return on physical capital is equal to the (real) risk-free rate R_t. This has important implications when the model is confronted to data in an estimation process like the one described later in the paper. Indeed, the expected return to capital will be linked to real activity, represented in the estimation process by data on GDP or aggregate investment and consumption. Further, the risk-free rate will typically be linked to short-term rates targeted by central banks or that on government bonds. An expression like (10) thus imposes a specific correlation between economic activity and interest rates and if this correlation is absent in the data used, the model will not be able to replicate it well. Introducing financial frictions, as shown below, makes (10) more flexible, potentially allowing it to better replicate data patterns.

2.2 Labour markets

Labour packers

Labour packers produce the composite labour input L_t by purchasing differentiated labour inputs $l_t(l)$ at price $W_t(l)$ from labour unions, where $l \in (0,1)$. These inputs are aggregated –"packed"– into a composite labour input L_t using the aggregation technology

$$L_t = \left[\int_0^1 l_t(l)^{\frac{\epsilon_w - 1}{\epsilon_w}} dl\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{11}$$

where ϵ_w is the elasticity of substitution between the differentiated labour types. This composite labour input is sold to intermediate-good producers (see below) at price W_t. Labour packers operated under perfect competition and profit maximisation leads to the following input demand for each labour type:

$$l_t(l) = \left(\frac{W_t(l)}{W_t}\right)^{-\epsilon_w} L_t. \tag{12}$$

Meanwhile, the zero-profit condition associated with the perfectly competitive nature of the market, combined to the constant-returns-to-scale technology (11), leads to the following price for the composite labour input L_t:

$$W_t = \left[\int_0^1 W_t(l)^{1-\epsilon_w} dl\right]^{\frac{1}{1-\epsilon_w}}. \tag{13}$$

Labour unions

Labour unions purchase homogenous labour services from households at market cost W_t^h; they are price-takers in that market. Next, they costlessly differentiate these labour services into heterogenous labour types $l \in (0,1)$, thus gaining market power. Further, the pricing decisions they must make is affected by a nominal rigidity à la Calvo (1983). More precisely, suppose that each labour union is able to re-optimise the price $W_t(l)$ for variety l only after having received a signal occurring with probability $1 - \xi_w$. If this signal is not received (probability ξ_w) the labour union cannot operate a full reoptimization but instead adjusts its price to aggregate inflation according to the following indexation rule:

$$W_t(l) = W_{t-1}(l)\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota_w}, \tag{14}$$

where ι_w measures the degree of indexation.

Consider a labour union l that has received the signal to reoptimize and denote its optimal choice by $W_t^*(l)$. In the context of (12), which represents the demand for its product, and the indexation rule (14), the optimization problem for setting $W_t^*(l)$ is the following:

$$\max_{W_t^*(l)} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s (\frac{\lambda_{t+s}}{\lambda_t})\, l_{t+s}(l) \left[\frac{W_t^*(l)}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\iota_w} - \frac{W_{t+s}^h}{P_{t+s}} \right] \tag{15}$$

where $(\beta)^s(\frac{\lambda_{t+s}}{\lambda_t})$ is the discount factor that labour unions apply to profits realized at time $t+s$.

The first-order condition associated with the labour unions' optimization problem entails

$$E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s (\frac{\lambda_{t+s}}{\lambda_t})\, l_{t+s}(l) \left[\frac{W_t^*(l)}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\iota_w} - \frac{W_{t+s}^h}{P_{t+s}} M_{w,t+s} \right] = 0; \tag{16}$$

where $M_{w,t} \equiv \frac{\epsilon_w}{\epsilon_w - 1} u_t^w$ is the gross wage mark-up with shock u_t^w assumed to follow a first-order autoregressive process with serial correlation ρ_w and innovation $\varepsilon_t^w \sim (0, \sigma_w^2)$.

Finally, the law of large numbers implies that every period a fraction $1 - \xi_w$ of labour unions reoptimize, while a proportion ξ_w set their price according to the rule (14); together these decisions lead to the following evolution for the aggregate price of the labour input W_t defined in (13):

$$W_t = \left[(1-\xi_w) W_t^*(l)^{1-\epsilon_w} + \xi_w \left(W_{t-1} (\frac{P_{t-1}}{P_{t-2}})^{\iota_w} \right)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}}. \tag{17}$$

2.3 Goods market

Final goods producers

Much like the *labour packers* described above, final goods producers purchase intermediate goods $y_t(r)$, $r \in (0,1)$ at price $p_t(r)$, and aggregate them to form the composite, final good Y_t using the aggregation technology

$$Y_t = \left[\int_0^1 y_t(r)^{\frac{\epsilon-1}{\epsilon}} dr \right]^{\frac{\epsilon}{1-\epsilon}}, \tag{18}$$

where ϵ measures the elasticity of substitution between intermediate goods. The composite good Y_t is sold at price P_t to households, capital producers and the government, under a perfectly competitive structure. Once again, the input demand $y_t(r)$ for each intermediate good obtains from the profit maximization problem and is

$$y_t(r) = \left(\frac{p_t(r)}{P_t} \right)^{-\epsilon} Y_t, \tag{19}$$

while the no-profit condition allows for the following definition for final-good price P_t:

$$P_t = \left[\int_0^1 p_t(r)^{1-\epsilon} dr \right]^{\frac{1}{1-\epsilon}}. \tag{20}$$

Retailers

Retailers behave similarly to the labour unions described above: they purchase homogenous intermediate goods, at price ϕ_t (measured relative to final-goods), and differentiate them costlessly, thus acquiring market power. As above, we assume that each retailer can re-optimise the price $p_t(r)$ only after receiving a random signal that occurs with probability $1-\xi_p$. If this signal is not received (probability ξ_p) the retailer does not reoptimize but modifies it price according to the indexation rule

$$p_t(r) = p_{t-1}(r)\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota_p} \tag{21}$$

where ι_p measures the degree of indexation.

The demand faced by retailers is drawn from (19). Considering this as well as the indexation rule in (21), a retailer having received the signal to re-optimize will set $p_t^*(r)$ in order to solve the following problem:

$$\max_{p_t^*(r)} E_t \sum_{s=0}^{\infty}(\beta\xi_p)^s(\frac{\lambda_{t+s}}{\lambda_t})y_{t+s}(r)\left[\frac{p_t^*(r)}{P_{t+s}}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\iota_p} - \phi_{t+s}\right], \tag{22}$$

with the associated first-order condition:

$$E_t \sum_{s=0}^{\infty}(\beta\xi_p)^s(\frac{\lambda_{t+s}}{\lambda_t})y_{t+s}(r)\left[\frac{p_t^*(r)}{P_{t+s}}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\iota_p} - \phi_{t+s}M_{p,t+s}\right] = 0; \tag{23}$$

where $M_{p,t} \equiv (\frac{\epsilon}{\epsilon-1})u_t^p$ is the gross price mark-up, which is affected by a shock u_t^p governed by an autoregressive process with serial correlation ρ_p and innovation $\varepsilon_t^p \sim (0, \sigma_p^2)$. Finally, the dynamics of the final good price P_t are similar to that of the wage W_t and are thus

$$P_t = \left[(1-\xi_p)p_t^*(r)^{1-\epsilon} + \xi_p\left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota_p}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{24}$$

Intermediate goods producers

Intermediate-good firms produce y_t by hiring capital services $u_t K_t$ from households at rate R_t^H and labour services L_t from labour packers at price W_t. These two inputs are combined using the standard Cobb-Douglas function

$$y_t = a_t(u_t K_t)^{\alpha}(L_t)^{1-\alpha}, \tag{25}$$

with α the capital share and a_t a productivity shock governed by a first-order auto-regressive process with coefficient ρ_a and innovation $\varepsilon_t^a \sim (0, \sigma_a^2)$. The usual first-order conditions for capital and labour inputs used apply, so that (recall that ϕ_t is the relative price of intermediate):

$$R_t^H = \phi_t\alpha(\frac{y_t}{u_t K_t}); \tag{26}$$

$$\frac{W_t}{P_t} = \phi_t(1-\alpha)(\frac{y_t}{L_t}). \tag{27}$$

2.4 Capital producers

Following much of the literature (Bernanke et al., 1999; Christiano et al., 2005; Brzoza-Brzezina and Kolasa, 2013) we assume that capital producers combine the stock of non-depreciated capital $(1-\delta)k_t$ with a quantity i_t of final goods and transform them into new units of the capital good, then sold to households in a competitive market at price q_t. This entails the following accumulation law for capital:

$$k_{t+1} = (1-\delta)k_t + x_t\left[1 - F(\frac{i_t}{i_{t-1}})\right]i_t, \tag{28}$$

where $F(\frac{i_t}{i_{t-1}})$ represents adjustment costs that punish large changes to investment and x_t is an investment-specific technology shock affecting the economy's ability to transform final goods into capital. Once again, this shock has an AR(1) structure with coefficient ρ_x and innovation $\varepsilon_t^x \sim N(0, \sigma_x^2)$. The optimal choice of capital producers leads to the following expression

$$1 = q_t x_t \left[1 - F(\frac{i_t}{i_{t-1}}) - F'(\frac{i_t}{i_{t-1}})(\frac{i_t}{i_{t-1}})\right] + \beta E_t \left[(\frac{\lambda_{t+1}}{\lambda_t}) q_{t+1} x_{t+1} F'(\frac{i_{t+1}}{i_t})(\frac{i_{t+1}}{i_t})^2\right] \qquad (29)$$

2.5 Monetary and fiscal policies

The monetary authority sets the (gross) nominal interest rate R_t^n by following the Taylor rule

$$ln(\frac{R_t^n}{R^n}) = \rho_i ln(\frac{R_{t-1}^n}{R^n}) + (1 - \rho_i) \left[\rho_\pi ln \frac{\Pi_t}{\Pi} + \rho_y ln \frac{Y_t}{Y}\right] + \varepsilon_t^r, \qquad (30)$$

where R^n, Π and Y are the steady-state values of the nominal interest rate, the gross inflation rate and aggregate output, respectively, while $\varepsilon_t^r \sim N(0, \sigma_r^2)$ is a monetary policy shock.[6] The standard interpretation of this rule is that it reflects the central bank's use of short-term nominal rates with the objective to minimize deviations of the inflation rate and production from their target values. The link between the nominal interest rate R_t^n and the real rate described above is established via Fisher's relation:

$$R_t = E_t(\frac{R_t^n}{\Pi_{t+1}}). \qquad (31)$$

On the fiscal side, we posit that government purchases of final goods every period are represented by g_t, financed via lump-sum taxation T_t imposed on households. These public expenditures are assumed to follow another auto-regressive process, with coefficient ρ_g and innovation $\varepsilon_t^g \sim N(0, \sigma_g^2)$. This streamlined view of fiscal policy is meant to focus on the aggregate-demand-shifting properties of government purchases.

2.6 Aggregation and market equilibrium

The model is closed by the following resource constraint :

$$Y_t = c_t + i_t + g_t + \upsilon(u_t) K_t \qquad (32)$$

which states that aggregate production is allocated to consumption expenditures, investment, government expenditures and costs related to changes in the utilisation rate of capital.

3 Two models with financial frictions

This section presents two model versions with financial frictions. They share the following key departure from the benchmark: households do not directly manage the economy's stock of physical capital and do not choose its utilization rate. Instead, a new class of economic agents –entrepreneurs– make decisions related to capital accumulation and its utilization rate; households now only indirectly indirectly these decisions, by financing part of entrepreneurs' purchases of capital. The two models do differ because a distinct agency problem affects the link between savings and capital allocation

[6]Note that we are assuming the shock ε_t^r has zero persistence, even though the interest rate itself will have significant persistence because of the form of (30).

in each case: the CSV framework from Bernanke et al. (1999) in the first model, and the costly enforcement environment used by Gertler and Karadi (2011) in the second.

As we show below, the key *practical* difference between these two environments lies in the dynamic link they imply between current leverage and the expected future spread between returns to capital and risk-free rates: while the financial contract derived from Bernanke et al. (1999) links current leverage to the one-period-hence expected spread, we show that Gertler and Karadi (2011) implies a link between current leverage and a distributed sum of all such future expected spreads.

In addition, we argue that it may not be crucial to specifically assign the agency problem as applying to entrepreneurs, as in Bernanke et al. (1999), or to financial intermediaries (Gertler and Karadi, 2011), especially when these models are used in applied setting that only use aggregate data, such as the medium-scale models developed in many central banks worldwide. We instead consider that each agency problem affects the link between households (the ultimate sources of loanable funds) and a combined entrepreneurial-intermediation block (the ultimate users of loanable funds). As such, the form of agency problem used by a modeler may not have strong sectoral implications and a finding that one friction is preferred to the other simply indicates that the dynamic structure implied by that friction matches available data better.[7]

3.1 Households

As mentioned above, households now do not make physical capital management decisions. They simply choose labour supply l_t, consumption c_t and savings through one-period bonds b_{t+1}, interpreted as lending to the combined Entrepreneurial/Banking block. To cover these expenditures, the representative household relies, as before, on labour income $\frac{W_t^h}{P_t}$, gross returns on financial assets (loans) $R_{t-1}b_t$, transfers T_t from the government and dividends Π_t from firms. Optimization requires that the choices for c_t, b_{t+1} and l_t maximise expected flows of discounted utilities in (3) under the following, updated budget constraint:

$$c_t + b_{t+1} \leq \frac{W_t^h}{P_t} l_t + R_{t-1} b_t + T_t + \Pi_t \tag{33}$$

First-order conditions associated with this problem are:

$$(c_t - hC_{t-1})^{-1} = \lambda_t; \tag{34}$$

$$\beta R_t E_t(\lambda_{t+1}) = \lambda_t; \tag{35}$$

$$l_t^{\phi} = \lambda_t \frac{W_t^h}{P_t}. \tag{36}$$

3.2 Entrepreneurs

Entrepreneurs are a new class of risk-neutral agents in the economy, who accumulate physical capital and manage its utilization. Specifically, they purchase new capital goods at the end of period t and, in period $t+1$, choose an utilisation rate and rent the resulting capital services to intermediate-good producers. Entrepreneurs' purchases of capital are financed by their own accumulated net worth (see below) and by lending originating from households' savings (the bonds b_t described above).

[7] Recall that by contrast, Villa (2013, 2016) makes a clear distinction between the first friction, hypothesized to apply to the entrepreneurial sector, and the second, assumed to relate to the banking sector, thus inviting sectoral implications for econometric results about which friction is preferred by the data.

To ensure entrepreneurial net worth never becomes sufficient to completely cover desired capital purchases, entrepreneurs are assumed to be finite-lived: an entrepreneur alive at period t survives with probability θ, constant and independent of history, which implies that a given's entrepreneur expected life is $\frac{1}{1-\theta}$. The fraction $(1-\theta)$ of entrepreneurs who exit the economy at the end of each period consume their accumulated net worth and at period $t+1$, a cohort of newly-born entrepreneurs enters the scene with a very small amount of net worth N_t^e.[8]

As of mid-period t, after having received payments related to current rental services from capital, a given entrepreneurs purchases the quantity of capital K_{t+1} for next period's use at price q_t. Total outlays are thus $q_t K_{t+1}$, which are financed by using accumulated net worth N_t and external finance $B_{t+1} = q_t K_{t+1} - N_t$ from the financial intermediary/collective household savers. Expected receipts in period $t+1$ are as follows: income $R^H_{t+1} u_{t+1} K_{t+1}$ from the rental of capital services to intermediate-good producers –with u_{t+1} the utilisation rate of capital and associated utilisation costs $\upsilon(u_{t+1})K_{t+1}$– as well as $q_{t+1}(1-\delta)K_{t+1}$, the value of non-depreciated capital. Overall, the future return to capital expected by the entrepreneur is thus

$$E_t(r^k_{t+1}) = E_t\left[\frac{R^H_{t+1}u_{t+1} - \upsilon(u_{t+1}) + (1-\delta)q_{t+1}}{q_t}\right] \tag{37}$$

Notice that as written, this expected return to capital is identical to (9), its definition when households own and operate the capital in the no-friction model. We now present the specific features of the two financial-friction models.

3.3 Financial Friction I : Costly State Verification

Both financial frictions imply an agency problem between households, who provide the bulk of the financing of capital purchases and the entrepreneurs, who use these funds to purchase and manage capital goods. The first formulation we use is the costly-state verification environment from Bernanke et al. (1999), which arises as follows. Entrepreneurial project returns are subjected to idiosyncratic risk, with the realized project return $\omega E_t(r^k_{t+1})$, where ω is a $i.i.d$ variable with mean 1 and cumulative distribution function $F(\omega)$; meanwhile, $E_t(r^k_{t+1})$ is the ex-ante aggregate return displayed in (37). Projects with relatively high realized returns, ie. $\omega \geq \overline{\omega}$ compel entrepreneurs to pay lenders back normally, whereas those with $\omega < \overline{\omega}$ will lead them instead to default on their obligations to lenders, with the cut-off value $\overline{\omega}$ determined endogenously. Since the idiosyncratic return is private, an incentive exists to declare default even when good results have obtained and as a result, defaulting leads to an automatic audit of the failed project. In such an event project managers (the entrepreneurs) receive nothing and lenders keep the project's recoverable value ie. $(1-\mu)\omega r^k_{t+1} q_t k_{t+1}$, where μ represents monitoring or auditing costs that the lender bears in order to recover value from a defaulted project.

Bernanke et al. (1999) integrate these features in a financial contract that maximizes the expected net return to the entrepreneur, subject to a participation constraint ensuring that households (the ultimate purveyors of loanable funds) receive the opportunity costs of the funds engaged in financing, which is the risk-free return R_t (recall the budget constraint (33)). The upshot of this contract is a positive relationship between leverage $q_t K_{t+1}/N_t$ achieved by a given entrepreneur over the internal funds invested in the project (the net worth N_t), on the one hand, and the expected

[8] Gertler and Karadi (2011) use a similar assumption and operationalize it by assuming that entrepreneurs are agents "on leave" from their larger household family, with a mandate to accumulate net earnings and transfer them back to their larger family when exiting the entrepreneurial sector.

premium in the return to capital $E_t(r^k_{t+1})$ over the riskless rate R_t, on the other, so we have

$$\frac{q_t K_{t+1}}{N_t} = \psi\left(E_t r^k_{t+1}/R_t\right),\ \psi'(.) > 0 \tag{38}$$

where the specific form of $\psi(.)$ results from parametric assumptions about the distribution of idiosyncratic shocks $F(\omega)$ and the auditing costs μ. Since $\psi'(.) > 0$, a higher expected return to installed capital, all things equal, increases the borrowing capacity of a given entrepreneur and the leverage that can be achieved over net worth N_t.[9] Note that up to a first-order approximation, (38) can be rewritten as

$$\hat{q}_t + \hat{k}_{t+1} - \hat{N}_t = \frac{1}{\nu}\left[\hat{r^k}_{t+1} - \hat{R}_t\right], \tag{39}$$

where a hatted variable expresses its deviation from steady-state and ν is the (inverse) of the elasticity of $\psi(.)$ with respect to the premium evaluated at steady state (Bernanke et al., 1999). For convenience, let us write (39) as the following:

$$\hat{q}_t + \hat{k}_{t+1} - \hat{N}_t = \phi^{BGG}\left[\hat{r^k}_{t+1} - \hat{R}_t\right], \tag{40}$$

where ϕ^{BGG} naturally equals $1/\nu$. Considering a range $\nu \in [0.04\ 0.10]$ has been used in the literature (Bernanke et al., 1999; De Graeve, 2008; Christensen and Dib, 2008; Villa, 2016), (40) implies values $\in [10\ 25]$ for ϕ_{BGG}. As such, this literature implicitly assumes that entrepreneurs' leverage is highly responsive to small disruptions in the capital return to risk-free rate spread: a 1% shock to that premium thus leads to a 20% spike in leverage according to the benchmark calibration of Bernanke et al. (1999) ($\nu = 0.05$). As shown below, the contract derived from the costly enforcement in Gertler and Karadi (2011) leads to very different dynamics.

Considering that the amount lent out to a given entrepreneur is $q_t K_{t+1} - N_t$; that successful entrepreneurs one-period hence pay back $E_t[r^k_{t+1}](q_t K_{t+1} - N_t)$ to financial intermediaries; and, finally, that a fraction $1-\theta$ of entrepreneurs exit at the end of the period and are replaced with ne ones, the following law of motion for the aggregate stock of entrepreneurial net worth obtains:

$$N_{t+1} = \theta\left[r^k_{t+1} q_t K_{t+1} - E_t[r^k_{t+1}]\left(q_t K_{t+1} - N_t\right)\right] + (1-\theta) N^e_t. \tag{41}$$

3.4 Financial friction II : Costly Enforcement

The second framework with financial frictions arises from Gertler and Karadi (2011). It posits the following problem of costly enforcement: after obtaining resources from the intermediary and purchasing $q_t K_{t+1}$ in newly-produced capital, an entrepreneur may choose to divert these resources towards a private project and abandon his loan engagements. Gertler and Karadi (2011) assume that in such an instance, lenders can only repossess a fraction $(1-\omega)$ of the project value, with the parameter ω related to institutional aspects of bankruptcy laws or to the practical ease by which values from bankrupt projects is realized. The cost of default, from the entrepreneur's point of view, is a permanent ban from credit markets and as such the decision to default will weigh the arbitrage between the long-term benefits of continued access to credit (a "charter value") versus the short term value of the diverted funds.[10]

In that context, the entrepreneur's decisions are as follows. At the end of period t, this en-

[9] See Bernanke et al. (1999) for details about the properties of $\psi(.)$.

[10] Note that modifying the period during which a defaulting borrower is banned from credit markets could change this arbitrage and, by extension, the quantitative properties of the contract derived from that arbitrage.

trepreneur purchases the quantity K_{t+1} of capital goods at price q_t, covering the expenses $q_t K_{t+1}$ with a mix of accumulated net worth N_t and external funds $B_t = q_t K_{t+1} - N_t$, whose cost are R_t, the opportunity costs of funds from the lending intermediaries having access to household savings. Considering that the project will obtain a return r^k_{t+1} next period, the entrepreneur considers the following law of motion for net worth:

$$N_{t+1} = r^k_{t+1} q_t K_{t+1} - R_t B_t = \left(r^k_{t+1} - R_t\right) q_t K_{t+1} + R_t N_t, \tag{42}$$

where the last equality illustrates how the ability to leverage net worth into large projects via external funds leads to excess returns. As we did above, we assume that entrepreneurs are finitely-lived, with probability $1-\theta$ of exiting the economy at every instance while θ is the probability of surviving. Considering that surviving entrepreneurs have the incentive to keep investing all returns in new projects to bring back the maximum income possible to their extended household family, the *expected* terminal-period net worth for a given entrepreneur is

$$V_t = E_t \sum_{s=0}^{\infty} (1-\theta)\theta^s (\beta)^{s+1} \left(\frac{\lambda_{t+1+s}}{\lambda_t}\right) \left[(r^k_{t+1+s} - R_{t+s}) q_{t+s} K_{t+s+1} + R_{t+s} N_{t+s}\right] \tag{43}$$

with respect to (42). In this expression, the quantity $\beta^{s+1}\lambda_{t+1+s}/\lambda_t$ reflects the fact that exiting entrepreneurs re-integrate a household "family". The quantity V_t represents the value, for an entrepreneur, of continued access to credit markets: as such the costly-enforcement problem discussed above implies that lenders will ration borrowers so to the point where an incentive to honor engagements and not default remains. This requires

$$V_t \geq \omega q_t K_{t+1}. \tag{44}$$

Gertler and Karadi (2011) demonstrate that V_t can be expressed using the following recursive formulation:

$$V_t = \nu_t q_t K_{t+1} + \eta_t N_t, \tag{45}$$

where

$$\nu_t = E_t \left[(1-\theta)\beta \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(r^k_{t+1} - R_t\right) + \beta\theta\nu_{t+1} \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(\frac{q_{t+1}K_{t+2}}{q_t K_{t+1}}\right)\right], \tag{46}$$

and

$$\eta_t = E_t \left[\beta(1-\theta) R_t \left(\frac{\lambda_{t+1}}{\lambda_t}\right) + \beta\theta\eta_{t+1} \left(\frac{\lambda_{t+1}}{\lambda_t}\right) \left(\frac{N_{t+1}}{N_t}\right)\right]. \tag{47}$$

By combining (44) holding at equality and (45), one obtains the following for maximum leverage allowed:

$$\frac{\eta_t}{\omega - \nu_t} = \frac{q_t K_{t+1}}{N_t} \tag{48}$$

This is *qualitatively* similar to (38) above, obtained with Bernanke et al. (1999)' CSV environment. In both cases, a rise in the expected return to installed capital increases the borrowing capacity for a given entrepreneur; ie. it entails an increase in the leverage $q_t K_{t+1}/N_t$ achieved over accumulated net worth (in 48, it works through an increase in ν_t). However, expression (48) is also quite different quantitatively: relative to (38), it entails that a weighted average of *all* expected future spreads between capital returns and risk-free rates are important in establishing current leverage. Indeed one can show that up to a first-order approximation, (48) takes the following recursive form

$$\hat{q}_t + \hat{k}_{t+1} - \hat{N}_t = \phi_1^{GK} \left[\hat{r^k}_{t+1} - \hat{R}_t\right] + \phi_2^{GK} \left[\hat{q}_{t+1} + \hat{k}_{t+2} - \hat{N}_{t+1}\right], \tag{49}$$

where the coefficients ϕ_1^{GK} and ϕ_2^{GK} depend on the model's calibrated structural parameters and steady state. An expression like (49) is *implicit* in work using the costly enforcement of Gertler and Karadi (2011) and solving the model using first-order solutions. However, ours is the first paper to explicitly develop and analyze this expression.[11]

Comparing (40) and (49) illuminates the key practical difference between the financial friction in Bernanke et al. (1999) versus the one in Gertler and Karadi (2011). While the former entails a well-known link between current leverage and the one-period-hence expected spread between capital returns and the risk-free rate, the latter delivers a forward-looking, recursive form for leverage that implicitly includes all future such spreads. In the literature using the latter type of frictions (Gertler and Karadi, 2011; Villa, 2016) values for ϕ_1^{GK} and ϕ_2^{GK} in the vicinity of 2.5 and 0.98, respectively, are commonly used. This implies that the contrast between (40) and (49) is quantitatively substantial; according to the former, leverage reacts solely, but substantially ($\phi^{BGG} \in [10\ 25]$), to the one-period-hence expected spread; meanwhile the latter implies that leverage responds much more modestly to next period's spread ($\phi_1^{GK} \approx 2.5$) but substantially to all future such premia, through the term $\phi_2^{GK} \approx 0.98$. Our empirical work below exploits this stark difference between the two models when analyzing which one is preferred by the data.

Finally, note that the aggregate level of entrepreneurial net worth evolves according to the law of motion

$$N_{t+1} = \theta \left[(r_{t+1}^k - R_t) \frac{q_t K_{t+1}}{N_t} + R_t \right] N_t + (1-\theta) N_{t+1}^e, \tag{50}$$

with the equilibrium solution for allowed leverage in (48) incorporated in (50).

4 Data and estimation strategy

Under rational expectations, a first-order approximate solution for each of the three model versions is computed using standard methods and takes the following state-space form:

$$\hat{s}_t = A\hat{s}_{t-1} + B\varepsilon_t,\ \varepsilon_t \sim N(0, \Omega) \tag{51}$$

and

$$\hat{o}_t = C\hat{s}_t, \tag{52}$$

where again hatted variables represent deviations relative to steady-state values, the vector s_t collects all state (pre-determined and exogenous) variables, o_t designates the vector of observable endogenous variables and ε_t denote perturbations to the state vector, incorporating the model's exogenous shocks. The matrices A, B, and C are non-linear functions of all model parameters and naturally depend on model specification.

As written, the system (51)-(52) supposes that all parameter values underlying the matrices A, B, C and D are known. A dynamic literature has emerged to provide methods whereby time-series data are used in conjunction with the solution (51)-(52) to estimate the parameters underlying these matrices. The present paper uses Bayesian estimation via the MCMC algorithm, as implemented by Dynare, to conduct the estimation.[12]

Our estimation uses the following set of variables for the vector o_t: real gross domestic product

[11] Details on how to obtain (49) are available from the authors.

[12] See An and Shorfheide (2007) and Fernández-Villaverde (2010) for overviews of Bayesian estimation of DSGE models via the MCMC. Other estimation methods employed in the DSGE context include full-information maximum likelihood (Ireland, 2004) or conditional moment matching (Christiano et al., 2005; Meier and Muller, 2006).

(which corresponds to y_t in the model), real private consumption expenditures (c_t), real private fixed investment (i_t), hours worked, the inflation rate (P_t/P_{t-1}) and the nominal interest rate (R_t^n). These data concern the U.S. economy and are downloaded from *FRED*. They are HP-filtered and cover the period 1966Q1-2008Q3. This sample corresponds to the pre-crisis period (the failure of Lehman Brothers happened in October 2008) and share their starting point with the data used by Smets and Wouters (2007). In this sense, the objective of our work is to compare the empirical performance of our model versions during "normal" times.

5 Results and Analysis

This section presents our estimation results. First, Section 5.1 discusses how a subset of the parameters are calibrated rather than estimated, a standard feature of the DSGE literature. Next, Section 5.2 displays the results of our benchmark estimation, which consists of a MCMC Bayesian estimation of all remaining parameters, for the three versions of the model. Section 5.3 presents our robustness analysis.

5.1 Parametrisation and Calibration

As mentioned above, a variable utilisation rate of installed capital is often introduced in DSGE models in order to loosen the otherwise tight relationship between the stock of capital and its rental rate. Above, we have described how utilizing capital at rate u_t entails costs of $\upsilon(u_t)$. In our empirical work, we follow Christiano et al. (2005) and assume

$$\upsilon(u_t) = \varrho_1(u_t - 1) + 0.5\varrho_2 (u_t - 1)^2 .$$

This functional form implies that a steady state with $u = 1$ is compatible with zero utilization costs provided that $\varrho_1 = R^H$; recall the first-order condition (8). We impose this and further denote $\zeta = \varrho_2/\varrho_1$; our Bayesian procedure below thus estimates the parameter ζ, from which a value for ϱ_2 can be recovered. Next, the adjustment costs affecting the process by which physical capital is accumulated, represented by the generic function $F(i_t/i_{t-1})$ above, are now specialized to

$$F(i_t/i_{t-1}) = 0.5\xi \left(\frac{i_t}{i_{t-1}} - 1\right)^2 ,$$

so that our Bayesian procedure estimates the value of ξ.

A subset of model parameters related to the production sector are calibrated to specific numerical values instead of being estimated, a common strategy in this literature when the data used in the subsequent estimation stage contain little information about them. In that context, Table 1 presents calibrated values for five such parameters, which are common to the three model versions. First, the discount factor β is fixed at 0.99, implying an (annualized) real interest rate equal to 4% in the steady-state. Next, the share of income allocated to capital α is equal to 0.3. The depreciation rate of capital, δ, is then set at 0.025, corresponding to an annualized rate of 10%. Finally, the elasticities of substitution in the goods market and in the labour market are both calibrated to the value 6, in order that price and wage net mark-ups of 20% obtain in the steady-state. These values are standard in the literature.

Next, key parameters related to the financial friction and the entrepreneurial sector of each model are assigned values. We impose that all model versions we examine share a common steady

Table 1: Calibrated Parameters: Production Sector

Parameter	Value
β, discount factor	0.99
α, capital income share	0.3
δ, depreciation rate	0.025
ϵ, elasticity of subst. in goods market	6
ϵ_w, elasticity of subst. in labour market	6

state in order to focus on the frictions' impact on the dynamic solution. To this end, the steady-state leverage of projet size over net worth (qk/N) is targeted to be 2, while the spread between returns to capital and the risk-free rate (r^k/R) is 200 basis points on an annualized basis (both values are in the middle of the range used by researchers working with financial friction models). Table 2 depicts how reaching these targets requires that in the CSV framework stemming from Bernanke et al. (1999), the parameter θ governing the accumulation of net worth in the entrepreneurial sector be set at 0.9851. By contrast, this requires $\theta = 0.9783$ in the costly enforcement mechanism from Gertler and Karadi (2011). Further, the parameter ν –recall expression (39)– is set to 0.05, following Bernanke et al. (1999) and Villa (2016). As discussed above, this value is in the middle of the range used in the literature and implies a value $\phi^{BGG} = 20$ in (40): current leverage displacements are thus associated with large changes in the premium of expected capital returns over the risk-free rate. In the costly enforcement framework, the fraction of funds that can be diverted by entrepreneurs (ω) is set to 0.348. Note that as mentioned above, the values of θ and ω thus chosen to meet targets related to the economy's steady-state also have implications for the dynamic solution: here they imply values of $\phi_1^{GK} = 2.41$ and $\phi_2^{GK} = 0.98$ once a first-order approximate solution for the leverage equation (49) is computed: as such, leverage is importantly linked to a distributed sum of future spreads of capital returns over risk-free rates.

Table 2: Calibrated Parameters: Financial Sector

Targets for Calibration	**Value**
$\frac{K}{N}$, Leverage Ratio	2
$\frac{r^K}{R}$, Capital Return to Risk-free Spread	200bp
Implied Parameter Values - Financial Friction 1	**Value**
θ, survival rate	0.9851
ν, elas. of leverage w.r.t. premium	0.05
Implied Parameter Values - Financial Friction 2	**Value**
θ, survival rate	0.9783
ω, fraction of funds entrepreneurs can divert	0.3479

5.2 Bayesian estimation: benchmark results

This subsection presents our benchmark results. We estimate the three versions of the model via Bayesian methods. Each model uses the calibrated parameter values in Table 1; additionally, the two financial-frictions models use the parameters described in Table 2. This leaves 22 that are estimated by each model: 8 parameters related to production and pricing (h, ϕ, ξ, ζ, ι_w, ξ_w, ι_p and

ξ_p), three related to the structure of the monetary policy reaction function (ρ_i, ρ_π and ρ_y), and eleven parameters governing the persistence (ρ_w, ρ_p, ρ_a, ρ_x, ρ_g) and volatility (σ_w, σ_p, σ_a, σ_x, σ_g and σ_r) of the six exogenous shocks.[13] It is worth emphasizing that the three models are estimated using the same, common set of six aggregate time-series described above. In that sense, the financial frictions affect estimation results only via expressions (40) and (49) and their impact on dynamics. In other words, a model with a financial friction will be preferred by the data only because a link like (40), say, allows to increase the computed model marginal likelihood.[14] Finally, note that the model with no financial frictions has no implications for entrepreneurial leverage and indeed has no role for entrepreneurs. This results in a steady state that is numerically different than the one common to the two financial-friction models. To ensure that we only study these models' ability to account for the *dynamics* of the data, we follow some of the literature (Bernanke et al., 1999; Meh and Moran, 2010) and apply the out-of-steady state dynamic solution of the no-financial model to the steady-state of the financial friction models.

Tables 6, 7 and 8, which we have relegated to the Appendix for conciseness, thus display the results the Bayesian estimates for the parameters related to the model with no financial friction, the model with the from Bernanke et al. (1999), and the model with a friction à la Gertler and Karadi (2011), respectively. Each table reports the prior distribution of the parameters and their posterior means as well as 90% confidence bands.

The priors used are broadly similar to those used in the literature estimating DSGE models via Bayesian methods (Smets and Wouters, 2003, 2007; Queijo von Heideken, 2009; Brzoza-Brzezina and Kolasa, 2013; Villa, 2016). First, a substantial degree of habit formation ($h = 0.7$) is present and labour supply is relatively elastic relative to the wage ($\phi = 2.0$). Further, the importance of nominal rigidities (ξ_p, ξ_w) and the extent to which indexing to past inflation is present (ι_p, ι_w) is assumed to be centered around 0.5. Turning to monetary policy, the priors related to the policy rule imply a fairly high degree of interest rate smoothing ($\rho_i = 0.75$), a response to inflation equal to the one originally advocated by Taylor ($\rho_\pi = 1.5$) and a modest response to output deviations from steady state ($\rho_y = 0.12$). Finally, the priors means for the shock processes' serial correlation and standard deviation parameters are set to 0.5 (auto-correlation) and 0.005 (standard deviations) with fairly diffuse priors, ie. large standard deviations around the prior means.

Table 6 reports results for the model without financial frictions. It shows that posterior distributions for some parameters have evolved significantly relative to priors. Notably, significant degrees of price and wage rigidity, as well as substantial degrees of indexation, have been updated upwards: the degree to which prices and wages are indexed to past values when new optimisation is not allowed (parameters ι_w and ι_p) notably see increases in means from prior (0.5) into the 0.75 range according to posteriors. In addition, the degree of interest-rate smoothing ρ_i is lower than suggested by the prior (0.68 relative to 0.75), the inflation response is higher (2.4 instead of 1.5), and the low response to output deviations in the prior has been further decreased to 0.08. Importantly, the volatilities in the wage and price mark-ups (σ_w and σ_p) are much higher than priors; this is contrast with other shock volatilities for which posterior means are slightly higher (the marginal efficiency of investment σ_x) or mostly unchanged from priors. Some important structural parameters have posterior ranges broadly unchanged from priors, notably, ξ and ζ, which index adjustments costs in investment and capacity utilization, respectively. Overall these results appear broadly consistent

[13] Recall that we have assumed the monetary policy shock ε_t^r has zero persistence.

[14] An alternative strategy by which models with financial frictions can better match data may be their ability to bring additional data, on leverage and premia for instance, to bear on the analysis. See De Graeve (2008) for a discussion and application of this strategy.

with those in the literature, eg. (Smets and Wouters, 2003, 2007).[15]

Next, Table 7 reports results for the Bernanke et al. (1999) financial-friction model. Some interesting results related to wage and price setting stand out: these parameters exhibit much lower values relative to their counterparts in the model without financial frictions. At the same time, the parameters related to price and especially wage rigidity (ξ_w) now have higher posterior means. Including financial frictions thus results in a revised appreciation of the process by which prices and wages evolve, with less importance given to indexation and more to actual rigidities. Table 7 also shows that findings about the monetary policy rule in the non-friction model above are largely confirmed: interest-smoothing is lower than prior, the response to inflation is higher and the response to output deviations continues to be very modest. Finally, some results related to the model's exogenous shocks are repeated, most notably the high estimated values for the volatility and serial correlation of the wage mark-up shock. Interestingly, Table 7 reports that parameters linked to the investment sector have been updated substantially: the model with financial frictions appears to give a higher importance to investment-specific technology shocks (as expressed by the higher posterior mean for the standard deviation σ_x for these shocks) and a higher posterior value for the structural parameter indexing investment-adjustment costs, now in the $\xi = 7.0$ range, compared to a prior of 4.0 and a posterior of 3.3 in Table 6. This interesting interaction between the presence of financial frictions and structural parameters linked to adjustment costs in the capital accumulation sector echoes results from Christensen and Dib (2008).

Table 8 finally reports estimates linked to the costly enforcement model à la Gertler and Karadi (2011). One can see from theses results that our appreciation for the model's nominal rigidities has changed again: the mean posterior estimates for the two parameters linked to indexation are now substantially lower than in both Table 6 and Table 7. By contrast, the parameter linked to adjustment costs for investment, (ξ), now has a much higher posterior mean than in the two previous tables of results. The parameters related to monetary policy remain in ranges similar to those obtained in the other cases, with the interest rate smoothing parameter notably seeing his posterior mean decline slightly to around 0.5. The main sources of volatility continue to arise from the shock to the marginal efficiency of investment (σ_x) and shocks to the wage mark-up shock (σ_w). As a recap, Tables 6-8 show that the financial friction importantly affect the Bayesian estimation of several model parameters, notably our appreciation of the overall stance of rigidity in price and wage-setting as well as the quantitative importance of real frictions, such as adjustment costs, in the capital accumulation sector.

5.3 Financial frictions and data marginal density

In addition to results about parameter estimates, Tables 6-8 report the (log) data marginal densities for each model. While the no-friction model reports a 4184.12 figure, the counterparts for the models with frictions are 4251.34 (BGG) and 4221.12 (GK), respectively. These results suggest that the data we use (i) prefer models with financial frictions to their no-friction counterpart, and (ii) that the CSV environment from Bernanke et al. (1999) appears to dominate the costly enforcement one from Gertler and Karadi (2011). However, note that the results obtained with the two financial friction models use *calibrated* values for the parameters expressing the frictions' impact on leverage (ϕ^{BGG} for the former and ϕ_1^{GK}-ϕ_2^{GK} for the latter). We now explore the extent to which an estimation strategy whereby these parameters are included in the Bayesian assessment (using their calibrated

[15] Smets and Wouters (2003) use linearly detrended data to conduct their analysis, while Smets and Wouters (2007) employ random walk processes. Such different differences can have important consequences for the interaction between model and data in the estimation process.

Table 3: Estimation of Parameters linked to Financial Frictions

	No Friction	*Models with Frictions*	
		Calibrated	Estimated
Panel A: BGG Model			
log(DMD)	4184.12	4251.34	4262.95
Estimate for ϕ^{BGG}		20	20 $\rightarrow$ 26.04
Panel B: GK Model			
log(DMD)	4184.12	4221.12	4285.68
Estimate for ϕ_1^{GK}		2.01	2.01 $\rightarrow$ 2.32
Estimate for ϕ_2^{GK}		0.980	0.980 $\rightarrow$ 0.977

Notes: (log) data marginal densities and calibrated as well as estimated values (priors, posteriors) for parameters related to the financial frictions.

values as priors) might modify this assessment.

To this end, Table 3 reports three estimates for the data's marginal densities in each of two panels, the top one being linked to the Bernanke et al. (1999) friction and the bottom one to the Gertler and Karadi (2011) framework. The three values for each model are obtained as follows. First, the data's marginal density when the friction is absent from estimation is reported in the left column of each panel. Next, the middle panel reports the data's marginal density when the friction has been included in the estimation, but with its value being set at the steady-state calibration stage. Finally, the right column of each panel depicts the result obtained when the friction has been added and included in the estimation stage, with a prior centered around the values calibrated when computing the steady state. The table reveals that as described above, data marginal densities are higher when the financial frictions are present, and that the Bernanke et al. (1999) appears to be preferred to the Gertler and Karadi (2011) framework when simply calibrated (4251.34 vs 4221.12, middle column of the tables).

However, the table also reports important results resulting from the inclusion of the parameters ϕ^{BGG} or ϕ_1^{GK}-ϕ_2^{GK} in the estimation stage. First, the data's marginal densities increase in both cases, but more so for the Gertler and Karadi (2011) framework, so that this friction now appears to be the one preferred by the data (4285.68 vs 4262.95). In addition, the posterior mean for ϕ^{BGG} (26) is substantially higher than its prior mean of 20, indicating a data preference farther in the range used in the literature than the one ($\phi^{BGG} = 20$) originally advanced by Bernanke et al. (1999). Finally, Bayesian estimation of ϕ_1^{GK}-ϕ_2^{GK} leads to a slight upwards revision to ϕ_1^{GK} as well as a slight downward revision to ϕ_2^{GK}. Overall these results strongly suggest that a flexible approach to the estimation of parameters related to financial frictions is a fruitful strategy likely to deliver important insights.

5.4 Sensitivity analysis

As indicated above, our benchmark estimation calibrates key parameters in order for the steady state of the two financial frictions' models to feature a 200 b.p. annualized spread between the return to capital and the risk-free rate in real terms (See Table 2). To test the robustness of our results to this target, Table 4 (BGG) and Table 5 (GK) report results obtained when the targeted steady-state spread is modified, to 100 or 300 basis points respectively, for the two types of financial

Table 4: Robustness Analysis: Steady-state Spread of Capital Return over Safe Rate
BGG Model

	No Friction	*Models with Frictions*	
		Calibrated	Estimated
Panel A: $r^k/R = 100$ *b.p.*			
log(DMD)	4081.21	4193.41	4224.36
Estimate for ϕ^{BGG}		20	$20 \rightarrow$ 37.88
Panel A: $r^k/R = 300$ *b.p.*			
log(DMD)	4238.67	4121.01	4275.22
Estimate for ϕ^{BGG}		20	$20 \rightarrow$ 33.90

Notes: (log) data marginal densities and calibrated as well as estimated values (priors, posteriors) for parameters related to the financial frictions.

Table 5: Robustness Analysis: Steady-state Spread of Capital Return over Safe Rate
GK Model

	No Friction	*Models with Frictions*	
		Calibrated	Estimated
Panel A: $r^k/R = 100$ *b.p.*			
log(DMD)	4080.25	3967.41	4018.78
Estimate for ϕ_1^{GK}		2.005	$2.005 \rightarrow$ 1.79
Estimate for ϕ_2^{GK}		0.975	$0.975 \rightarrow$ 0.970
Panel A: $r^k/R = 300$ *b.p.*			
log(DMD)	4238.67	4220.49	4193.40
Estimate for ϕ_1^{GK}		2.015	$2.015 \rightarrow$ 2.006
Estimate for ϕ_2^{GK}		0.985	$0.985 \rightarrow$ 0.998

Notes: (log) data marginal densities and calibrated as well as estimated values (priors, posteriors) for parameters related to the financial frictions.

frictions.

First, Table 4 shows that results obtained using the Bernanke et al. (1999) friction are very consistent with respect to that spread. Indeed, including the friction always results in an increase in the ability of the model to replicate the data (increases in the data's marginal density), especially when the parameter ϕ^{BGG} is estimated via Bayesian methods. In addition, the Bayesian procedure consistently pulls estimates for ϕ^{BGG} away from the prior mean of 20 and towards values that are significantly higher, up to almost 34 for example when the spread is 300 basis points.

Table 5 shows, however, that results are not as robust when the Gertler and Karadi (2011) friction is used. Indeed, the table shows that both for the low-spread and high-spread calibrations, neither the calibrated nor the estimated version of the model is capable of fitting the data better than the no-friction version of the model. Considering how much the friction-version of the model helped increase performance in the benchmark case with a 200 b.p. spread (See Table 3) this lack of consistency should be a signal that further empirical work on the Gertler and Karadi (2011) friction is necessary. In addition, further sensitivity analysis related to the steady state value of leverage could be undertaken.

6 Conclusion

This paper compares three estimated versions of the Smets and Wouters (2007) model: the first, used as a benchmark, includes no financial friction, while the second and third embed the CSV friction from Bernanke et al. (1999) and the costly enforcement problem of Gertler and Karadi (2011), respectively. Results indicate that the presence of financial frictions improves marginal densities and that the friction à la Gertler and Karadi (2011) may be preferred by the data.

Throughout, the paper argues that applied research on financial frictions' models can usefully concentrate on the practical quantitative implications of these frictions, that the specific location of the financial friction on saver-intermediary-borrower financing chain may not be crucial for medium-scale macroeconomic models and, finally, that estimation may benefit from proceeding in a manner that decouples computations related to steady states with those linked to the economy's dynamic solution around that steady state.

Additional research along in that direction might endeavour to encompass other financial frictions environments, such as those from Kiyotaki and Moore (1997) and Iacoviello (2005) to identify their implication for the leverage equation, or modify the Gertler and Karadi (2011) at the margins, by changing the period during which defaulting borrowers are banned from credit markets for instance, which again would influence the dynamic structure of the leverage equation.

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A Benchmark Estimation Results

Table 6: Model with no financial friction

	Prior			Posterior		
Parameters	Distrib.	Mean	Std. Dev.	10%	Mean	90%
Production and Pricing Parameters						
h	beta	**0.70**	0.1	0.704	**0.727**	0.742
ϕ	norm	**2.0**	0.75	1.143	**1.271**	1.409
ξ	norm	**4.0**	1.5	3.113	**3.323**	3.512
ζ	norm	**0.20**	0.075	0.248	**0.262**	0.275
ι_w	beta	**0.50**	0.15	0.694	**0.726**	0.761
ξ_w	beta	**0.50**	0.1	0.578	**0.609**	0.650
ι_p	beta	**0.50**	0.15	0.711	**0.744**	0.771
ξ_p	beta	**0.50**	0.1	0.715	**0.742**	0.761
Monetary Policy Parameters						
ρ_i	beta	**0.75**	0.1	0.669	**0.681**	0.691
ρ_π	norm	**1.50**	0.25	2.282	**2.356**	2.430
ρ_y	norm	**0.12**	0.05	0.077	**0.082**	0.087
Shock Parameters						
ρ_w	beta	**0.50**	0.2	0.996	**0.998**	0.999
ρ_p	beta	**0.50**	0.2	0.237	**0.267**	0.294
ρ_a	beta	**0.50**	0.2	0.655	**0.707**	0.753
ρ_x	beta	**0.50**	0.2	0.835	**0.885**	0.941
ρ_g	beta	**0.50**	0.2	0.782	**0.827**	0.886
σ_w	invg	**0.005**	2.0	0.031	**0.036**	0.041
σ_p	invg	**0.005**	2.0	0.035	**0.043**	0.052
σ_a	invg	**0.005**	2.0	0.003	**0.003**	0.003
σ_x	invg	**0.005**	2.0	0.008	**0.009**	0.010
σ_g	invg	**0.005**	2.0	0.007	**0.007**	0.008
σ_r	invg	**0.005**	2.0	0.002	**0.003**	0.003
Resulting log data marginal density:				**4184.12**		

Table 7: Model with the Costly State Verification friction (Bernanke et al., 1999)

	Prior			Posterior		
Parameters	Distrib.	Mean	Std. Dev.	10%	Mean	90%
Production and Pricing Parameters						
h	beta	**0.70**	0.1	0.605	**0.628**	0.650
ϕ	norm	**2.0**	0.75	1.885	**2.339**	2.782
ξ	norm	**4.0**	1.5	6.104	**6.598**	7.078
ζ	norm	**0.20**	0.075	0.156	**0.186**	0.219
ι_w	beta	**0.50**	0.15	0.380	**0.425**	0.471
ξ_w	beta	**0.50**	0.1	0.799	**0.852**	0.907
ι_p	beta	**0.50**	0.15	0.084	**0.148**	0.217
ξ_p	beta	**0.50**	0.1	0.550	**0.606**	0.658
Monetary Policy Parameters						
ρ_i	beta	**0.75**	0.1	0.618	**0.648**	0.676
ρ_π	norm	**1.50**	0.25	2.01	**2.10**	2.18
ρ_y	norm	**0.12**	0.05	0.000	**0.015**	0.029
Shock Parameters						
ρ_w	beta	**0.50**	0.2	0.987	**0.993**	0.999
ρ_p	beta	**0.50**	0.2	0.630	**0.690**	0.760
ρ_a	beta	**0.50**	0.2	0.901	**0.958**	0.993
ρ_x	beta	**0.50**	0.2	0.506	**0.585**	0.661
ρ_g	beta	**0.50**	0.2	0.690	**0.785**	0.921
σ_w	invg	**0.005**	2.0	0.032	**0.039**	0.046
σ_p	invg	**0.005**	2.0	0.007	**0.010**	0.012
σ_a	invg	**0.005**	2.0	0.002	**0.003**	0.003
σ_x	invg	**0.005**	2.0	0.028	**0.032**	0.036
σ_g	invg	**0.005**	2.0	0.005	**0.006**	0.007
σ_r	invg	**0.005**	2.0	0.002	**0.003**	0.003
Resulting log data marginal density:				**4251.34**		

Table 8: Model with the Costly State Verification friction (Gertler and Karadi, 2011)

	Prior			Posterior		
Parameters	Distrib.	Mean	Std. Dev.	10%	Mean	90%
Production and Pricing Parameters						
h	beta	**0.70**	0.1	0.599	**0.613**	0.621
ϕ	norm	**2.0**	0.75	0.000	**0.252**	0.434
ξ	norm	**4.0**	1.5	3.224	**3.530**	3.997
ζ	norm	**0.20**	0.075	0.126	**0.138**	0.147
ι_w	beta	**0.50**	0.15	0.623	**0.652**	0.685
ξ_w	beta	**0.50**	0.1	0.301	**0.328**	0.349
ι_p	beta	**0.50**	0.15	0.225	**0.271**	0.317
ξ_p	beta	**0.50**	0.1	0.555	**0.581**	0.606
Monetary Policy Parameters						
ρ_i	beta	**0.75**	0.1	0.554	**0.569**	0.593
ρ_π	norm	**1.50**	0.25	2.06	**2.11**	2.17
ρ_y	norm	**0.12**	0.05	0.070	**0.080**	0.095
Shock Parameters						
ρ_w	beta	**0.50**	0.2	0.995	**0.997**	0.999
ρ_p	beta	**0.50**	0.2	0.351	**0.378**	0.397
ρ_a	beta	**0.50**	0.2	0.986	**0.992**	0.999
ρ_x	beta	**0.50**	0.2	0.497	**0.529**	0.561
ρ_g	beta	**0.50**	0.2	0.778	**0.812**	0.858
σ_w	invg	**0.005**	2.0	0.011	**0.013**	0.015
σ_p	invg	**0.005**	2.0	0.011	**0.013**	0.015
σ_a	invg	**0.005**	2.0	0.002	**0.003**	0.003
σ_x	invg	**0.005**	2.0	0.015	**0.017**	0.020
σ_g	invg	**0.005**	2.0	0.006	**0.007**	0.008
σ_r	invg	**0.005**	2.0	0.003	**0.003**	0.003
Resulting log data marginal density:				**4221.12**		